The Constructivist Teaching Experiment in Mathematics Education Research*

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Running Head: Constructivist Teaching Experiment

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The Soviet-style teaching experiment has a long history in pedagogical research in the Soviet Union, but is somewhat new on the scene in the U.S. Kantowski[1] argued that the teaching experiment is a viable supplement to traditional research methodologies as employed in the U.S. to date. This paper will end in agreement with Kantowski, but for reasons other than those she gave.

The purpose of this paper is to examine the teaching experiment vis-a-vis classical methodologies, showing how the teaching experiment may be of practical use in overcoming epistemological difficulties inherent in more classical methods of pedagogical research. It will be shown that classical methods can make sense only if the researcher posits, tacitly or explicitly, (1) the predominance of the students' environment as a determiner of their behavior and (2) that the students' behavior is structurally determined by the structure of their environment. It will be argued that neither position is epistemologically tenable, and that both positions are not only misleading, but unproductive when the criterion is the understanding of the students' mathematical learning. The teaching experiment as employed by the Soviet's, and as employed by many American researchers, will also be shown to suffer from many of the problems affecting classical methodologies, and that a remedy can come only by way of a paradigm shift in pedagogical research in mathematics education.

The Teaching Experiment

The general characteristics of the Soviet-style "teaching experiment" have been well explicated elsewhere (Kantowski, ; Menchiskaya, 1969; Kantowski, Steffe, Lee, & Hatfield, 1978). Briefly they are:

(1) an orientation toward uncovering processes by which students learn school subject matter;

(2) the longitudinal nature of the investigation;

[1] Kantowski, M.G., Steffe, L.P., Lee, K.S., & Hatfield, L.H. The Soviet "teaching experiment": its role and usage in American research. Symposium presented at the annual meeting of the National Council of Teachers of Mathematics, San Diego, 1978.

(3) intervention by the researcher into the students' learning processes;

(4) the constant interaction between observations gathered up to the current point of the investigation and the planning of future activities in the investigation; and

(5) data that is qualitative rather than quantitative; whenever quantitative data is gathered it is used mainly in a descriptive manner.

This section will focus on another aspect of the teaching experiment that has heretofore been alluded to in the literature, but never fully addressed; namely, the epistemological considerations one must make when contemplating the teaching experiment as a research methodology. While the following remarks may well apply to any subject matter area, they will be couched in terms of mathematics education, since this is the area of our own interest and specialty.

Classical methodology

Traditionally, mathematics education research in the U.S. has focused largely on isolating variables in the students' environment that play an appreciable role in the way they learn mathematics. This approach has been used in research on effective teaching (Dossey, 1976), advance organizers (Romberg & Wilson, 1973), and the effect of subject matter organization (Gagne, 1962). The experimental methodologies employed in studies like these have their roots in what might be called the "agriculture paradigm." That is, the researcher selects one or more samples that, with some assurance, represent the target population and then subjects them to various treatments. The effect of one treatment is then compared to the effects of others with the intention of specifying differences between or among them. The treatments themselves arise from the researcher's logical analysis of the students' environment -- that is, from the researcher's attempt to make sense of the various possible factors in the students' environment that, through manipulation, might cause a corresponding variation in the target population. The manipulation of the variables through treatments is apparently an attempt by the researcher to observe the effects of various environments upon the students' behavior.

This is a seemingly natural way to approach problems of pedagogical theory building, and it is one advocated by Campbell and Stanley (1966). They state:

> By experiment we refer to that portion of research in which variables are manipulated and their effects upon other variables are observed... Insofar as the designs discussed in the present chapter become complex, it is because of the intransigency of the environment: because, that is, of the lack of complete control. (1966, p.1)

The experimental designs that Campbell and Stanley discuss arise primarily in response to two concerns: that the observed effects in an experiment are a result of the experimental treatments and not of variables extraneous to the experiment, and that the treatments, when given in other environments (i.e., in the context of other variables) will produce similar effects. Within the classical paradigm, then, a research program in experimental pedagogy is a search for lineal cause-effect relationships between the students' environment and their behavior. But there remains a problem -- what kind of thing is a "cause" when one is speaking of "causal relationships" between an "environment" and a cognizing organism? This question brings us to the truly epistemological difficulties inherent in mathematics education research.

Environments

The above passage ended with a question: What is sought in the search for causal relationships between students' behavior and their environment? Before examining this question it will help to simplify matters somewhat. First, we'll restrict ourselves to considering a single student (we could just as well speak of a "cognizing organism", but we'll leave the reader with some context); second, we'll consider what we mean by his "environment;" and third, we'll speak of the possible relationships that may exist between the two. Also, the reader is cautioned to take note that the following comments will always be made in the context of a researcher examining a phenomenon from a theoretical perspective.

When a researcher observes a phenomenon involving two or more items in his field of experience, and one of the items seems to be affected by the others, his first and foremost objective as a scientist is to postulate a lineal causal relationship between them. Such is the case with a physicist who observes the expansion of metals when they are exposed to a flame, or with a behavioral psychologist who observes the effects of drill on a pupil's behavior when recalling sums when the addends are presented on flash cards. In both of these cases the scientist is bent on isolating factors in (or aspects of) the environment of the item of interest which bear a relationship expressible in the form “If (conditions), then (consequence).” As it turns out, physicists have experienced a great deal of success in finding such relationships -- even ones expressible in the form of mathematical functions. Behavioral psychologists, on the other hand, have been far less successful than their counterparts in physics. It would seem a reasonable conjecture to maintain that they will never be likely to experience the same degree of success found with the physicist. Why is this so? Simply, because any child is far less predictable than a bar of metal. In fact, a child is unpredictable to the extent that the researcher attributes some form of information processing capability to him. That is, he assumes that the child gains information from his environment and then does something with it -- the product of the “something he does” is then the determiner of any behavior that the behavioral psychologist observes. But, having made such an assumption about that unpredictable kid, the researcher has lost any hope of building a lineal cause-effect model of his behavior! The connection between the child’s “environment” and his behavior breaks down just at the point where observation is impossible for the scientist -- at the child’s experiential interface. The behavioral psychologist indirectly avoids this problem by viewing the child as a “black box” about which nothing is known, but which nevertheless is “connected” with its environment in such a manner that its behavior is a direct result of environmental input. The main point to be drawn here is not that the behavioral psychologist ignores the information processing capability of the child, but that he continues to posit lineal cause-effect relationships between the child’s environment and the child’s behavior. B. F. Skinner (l972) has made one of the most explicit statements of the relationship between a cognizing organism’s environment and the organism itself to be found in behavioral psychology:

> Science…has simply discovered and used subtle forces which, acting upon a mechanism, give it the direction and apparent spontaneity which make it seem alive. (Skinner, 1972, p. 3)

Such a stance is not unique to behavioral psychology. Herbert Simon, foremost among information processing theorists, makes very much the same assumption, albeit he expresses it within a much different context than that in which Skinner works. Simon acknowledges the information processing capabilities of the human organism and also the apparent independence of man's "outer environment." However, he takes as a basic tenet of his psychology the fact that it is the outer environment of man that is the primary factor in determining his behavior. He states:

> A man, viewed as a behaving system, is quite simple. The apparent complexity of his behavior over time is largely a reflection of the complexity of the environment in which he finds himself. (1969, p. 25)

The emphasis that these schools of thought, as represented by the above passages, place upon the child's environment in explaining his behavior, introduces a problem that heretofore was largely obscured. This is the problem of determining the nature of the child's environment. To the educational researcher, the child's environment is everything in the researcher's field of experience that is not the child. The child's environment, at least to the researcher, is then a function of the researcher's way of viewing the world! The environment that the researcher sees is wholly inaccessible to the child, since it "comes from the researcher," so to speak. A simple, but by no means trivial, example is the researcher that is investigating the effects of the use of "manipulatives" in teaching addition to first-graders. When he places some checkers, say, on the table in front of the child, the researcher sees a well-defined collection, and possibly sees a specific number of checkers. These things are there for the researcher and hence they, as well as anything that he does with them, are in the child's environment. But what does the non-conserving child see? Certainly, not the same thing as does the researcher. Do the researcher's actions upon the checkers have the same meaning for the child as they do for the researcher? Again, certainly not. What, then, is the "environmental input" that "causes" any behavior that the researcher might see as a manifestation of the child's "information processing?" What "information" has been processed? The behavioral psychologist and information processing theorist that tacitly assume their reality to coincide with the child's have no hope of answering these questions, because, to them, they have never arisen. A realist epistemology forbids the consideration of such problems.

We do not mean to pick exclusively on behavioral psychology and information processing theory. Many developmental psychologists speak of properties, constructs, and relationships as existing "out there" -- in the "real world" -- for all to see if only their vision is clear (see, for instance, Gelman & Gallistel, 1978; Saxe, 1978). Cognitive development, to them, is something like a process of defogging one’s cognitive horizons -- as the mist clears we see reality as it is. Such a view, however, places us in an epistemological quagmire. If reality is “out there”, then how can we possibly come to know it when we are limited to experiencing only that which passes through the filter of our experiential interface?

Constructivism

One school of thought that fully addresses this problem and which dates at least from Bishop Berkeley (1610), and aspects of which were further developed by Kant, Poincare´, Dewey, Bridgeman, and Piaget to name a few, is the school of thought known today as “constructivism.” This section is not intended to give a full account of constructivism. The interested reader may refer to any of von Glasersfeld (1974, 1978a, 1978b), Piaget (1954), or Powers (1973a, 1973b) for more information. It is sufficient at this time merely to note that in its most radical form, any cognizing organism is viewed as building its own reality out of the very items that pass through its experiential interface. Of course, in this context “experiential interface” and “items” can have meaning only if we are speaking in observer language (MacKay, 1969). As von Glasersfeld (1974) notes, constructivism solves the problem of the relationship between environment and organism by simply acknowledging the inaccessibility (to us) of the child’s environment as seen from the child’s point of view, while at the same time acknowledging that we have no grounds for assuming it is the same as one’s own.

> We call this school of constructivism “radical” because it holds that the knower’s perceptual (and conceptual) activity is not merely one of selecting or transforming cognitive structures by some means of interaction with “existing” structures, but rather a constitutive activity which, alone, is responsible for every type or kind of structure an organism comes to “know.” (von Glasersfeld, 1974, p. l0)

Constructivist Experimental Pedagogy

The notion of explanation is an interesting one. In the philosophy of science, explanation is seen as always involving some form of causation (Nagel, 1965). Kerlinger & Pedhazur (1973) devote a chapter of their statistical methods book to techniques for assessing the viability of an explanation via correlational analyses. Campbell & Stanley (1966) address, throughout their book, problems the researcher faces in building an explanatory theory and techniques that may be used to alleviate them. In both cases, as in most in the field of experimental design and analysis, the emphasis is on experimental effects as measured in samples from a population. In this tradition, experimental pedagogues in mathematics education have focused mainly on the effects of treatments as measured by group performance. They have not attended, by and large, to the problems of how individual students learn mathematics. As Davis (1966) notes, pedagogical research in mathematics has been written for administrators. It bears little practicality for the person who must diagnose deficiencies and prescribe treatments for individual students. But as soon as attention is focused upon the individual student, the epistemological problems addressed above must be dealt with methodologically. Since the epistemological problems recommend constructivism as a point of view in experimental pedagogy, and since constructivism is concerned mainly with the possibility and nature of cognition by a child, it seems natural that the teaching experiment be used as a vehicle for constructivist experimental pedagogy.

The discussion under “Environments”, above, may seem to imply that a constructivist viewpoint militates against rational attempts to influence either the nature or rate of children’s intellectual growth. Since the child’s environment must be assumed to differ radically from the adult’s (i.e., respectively ours), and since that environment is wholly unknowable to us, we have no rational means to manipulate it. This is a serious objection, and it deserves consideration. Our response is that we still have rational recourse, but it is available only after we achieve a paradigm shift and consider not the child’s environment, but the child’s constructions. By building models to describe the nature of the constructive process, whether they be content-specific or general, it is possible to hypothesize not only what the world “looks like” through the child’s eyes, but ways in which the child comes to a particular point in his intellectual growth, how he got there, and ways that he might get there more efficiently. Only then may we, as educational researchers, select particular variables in our (respectively personal) environments to manipulate in hopes of generating theories which possess at least a modicum of explanatory and predictive power. But, to build such models individual students must be viewed in their dynamic aspects of construction. This may be done by employing the teaching experiment as an experimental methodology.

The "Teaching" Part of the Teaching Experiment

The Soviet-style teaching experiment as a methodology does not in and of itself resolve the epistemological problems addressed above. The question still remains as to how sense can be made, within a constructivist framework, of the researcher's activity as a "teacher" in a teaching experiment, when it has been acknowledged that the "teaching" that the researcher sees as being in the child's environment may have no counterpart in the child's environment from the child's point of view. If it is not possible to resolve this problem to our satisfaction, then, epistemologically speaking, we are no better off using the teaching experiment as an experimental methodology than we were when employing more classical ones.

An attempt to resolve these difficulties will be made by appealing to information theory. The reader is cautioned that, here, the term "information theory" has a much different designation than that normally found today in mathematics or communication engineering, or as in some of the recent literature in problem solving (cf. Kulm & Days, 1979). Information theory in these areas is a direct descendant of communication theory as developed by Shannon (1949) and Wiener (1948/1961), and does not deal with information per se, but with the quantification of the amount of information that can be conveyed through a communication system. It does not deal with questions of meaning (Shannon, 1949, p. 3). Information theory in the present usage, developed largely by MacKay (1949), subsumes communication theory and its modern derivatives in its attempt to explain the possibility of two systems communicating. It is the latter field that has produced a model that may be used to give an epistemologically consistent view of teaching.

Very briefly, a cognizing organism builds what might be called a "cognitive map" (MacKay, 1966, 1969), or a model of "reality" as seen from its side of its experiential interface. Part of that model includes things that behave in very unpredictable manners and which turn out to seem very difficult to control relative to other items in its field of experience. The human, as a cognizing organism, is capable of attributing a model building (i.e., "thinking") capability to these unpredictable objects (e.g., other humans) and it builds models of the other human's models (that is, it tries to understand what the other human intends). Of course this is all done, in observer language, at the level of the intuition. An observer of two humans (A and B) would see them each with a cognitive map which includes a model of the other's cognitive map. An attempt at communication between the two would be seen as an attempt on the part of, say, A to indicate to B an alteration that A wishes B to make in his cognitive map of reality, as dictated by A's model of B's cognitive map. This last qualifier is necessary, as A may (again, in observer language) have an invalid model of B's cognitive map.

To characterize the process of teaching, substitute "teacher" for "A and "student" for "B" in the characterization of the process of communication given above. The teacher has a model of a portion of the student's cognitive map and attempts to communicate to the student certain alterations to be made in it. As observers of this process, we see two necessary conditions for success: (1) the message as sent by the teacher must be assimilable by the student; and (2) the student must accommodate to the message. For these two conditions to be satisfied, and satisfied in the manner that the teacher intends, the teacher's model of the student's cognitive map must satisfy two further conditions -- it must have some validity, and it must specify means that the student has available to him to make the intended alterations. The art of teaching comes into existence when the alteration involves a major change in the student's cognitive map -- one which is achievable only by focusing over a period of time upon the means that the student has available to him to make the change.

We ask the reader to note that this view of teaching does not assume a lineal cause-effect relationship between what the teacher does ("environment") and what the student does. Not only is there a break in the connection between student and teacher at the student's experiential interface, there is also a break at the teacher's experiential interface. There are at least two models at play in any attempt at meaningful communication between human beings!

This view of learning and teaching, while rich in potential explanatory power, places a great burden on the researcher who is contemplating the use of a teaching experiment as a research methodology. Not only must he think in terms of a model of what the student sees himself (and the teacher) doing, he must also perform the ultimate act of decentration and think in terms of a (necessarily incomplete) model of what he sees himself doing in relation to the child.

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