Concrete Materials and Teaching for Mathematical Understanding†

Patrick W. Thompson

Center for Research in Mathematics and Science Education

San Diego State University

Running Head: Concrete Materials

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Learning without thought is labor lost.
— Confucius

An experience is not a true experience until it is reflective.
—John Dewey

Today there seems to be common agreement that effective mathematics instruction in the elementary grades incorporates liberal use of concrete materials. Articles in *The Arithmetic Teacher* no longer exhort us to use concrete materials, nor did the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics 1991) include a standard on the use of concrete materials. The use of concrete materials seems to be assumed unquestioningly.

My aim in this article is to reflect on the role of concrete materials in teaching for mathematical understanding, not to argue against their use, but instead to argue for using them more judiciously and reflectively. Our primary question should always be, “What, in principle, do I want my students to *understand*?” It is too often, “What shall I have my students learn *to do*.” If we can only answer the second question, then we have not given sufficient thought to what we hope to achieve by a particular segment of instruction or use of concrete materials.

Research on the Use of Concrete Materials

The use of concrete materials has always been intuitively appealing. The editors of a turn-of-the-century methods text stated, “Examples in the concrete are better for the student at this stage of his development, as he can more readily comprehend these” (Beecher and Faxon 1918, p. 47, as quoted in ; McKillip and others 1978). Their appearance accelerated in 1960's, at least in the United States, with the publication of theoretical justifications for their use by Zolton Dienes (1960) and by Jerome Bruner (1961).

A number of studies on the effectiveness of using concrete materials have been conducted since Dienes' and Bruner's publications, and the results are mixed. Fennema (1972) argued for their use with beginning learners while maintaining that older learners would not necessarily benefit from them. Suydam and Higgins (1977) reported a pattern of beneficial results for all learners. Labinowicz (1985) described the considerable difficulties had by his study's middle and upper primary students in making sense of base-ten blocks. Fuson and Briars (1990) reported astounding success in the use of base-ten blocks in teaching addition and subtraction algorithms. Thompson (1992) and Resnick and Omanson (1987) reported that using base-ten blocks had little effect on upper-primary students' understanding or use of their already-memorized whole-number addition and subtraction algorithms. Wearne and Hiebert (Hiebert, Wearne et al. 1991) report consistent success in the use of concrete materials to aid students' understanding of decimal fractions and decimal numeration.

These apparent contradictions probably are due to aspects of instruction and students' engagement to which studies did not attend. Evidently, just using concrete materials is not enough to guarantee success. We must look at the total instructional environment to understand effective use of concrete materials—especially teachers' images of what they intend to teach and students' images of the activities in which they are asked to engage.

Seeing Mathematical Ideas in Concrete Materials

It is often thought, for example, that an actual wooden base-ten cube is more concrete to students than is a picture of a wooden base-ten cube. As objects this certainly seems true. But to students who are still constructing concepts of numeration, the “thousand-ness” of a wooden base-ten cube often is no more concrete than the “thousand-ness” of a pictured cube (Labinowicz 1985). To understand the cube (either actual or pictured) as representing a numerical value of one thousand, students need to create an image of a cube that entails its relations to its potential parts (e.g., that it can be made of 10 blocks each having a value of one hundred, 100 blocks each having a value of ten, or 1000 blocks each having a value of one). If their image of a cube is simply as a big block named “thousand,” then there is no substantive difference to students between a picture of a cube or an actual cube—the issue of concreteness would be immaterial to their understanding of base-ten numeration. This is not to say that, to students, there is never a substantive difference between pictures and actual objects. Rather, it says only that concrete materials do not automatically carry mathematical meaning for students. There *can* be a substantive difference between how students experience actual materials and how they experience depictions of materials, but the difference resides in how they are used. I will return to this later.

To see mathematical ideas in concrete materials can be challenging. The material may be concrete, but the idea you intend that students see is not in the material. The idea you want your students to see is in the way you understand the material and in the way you understand your actions with it. Perhaps two examples will illustrate this point.

A common approach to teaching fractions is to have students consider a collection of objects, some of which are distinct from the rest, as depicted in Figure 1. The collection depicted in Figure 1 is certainly concrete. But, what might it illustrate to students? Three circles out of five? If so, they see a part and a whole, but not a fraction. Three-fifths of one? Perhaps. But depending on how they think of the circles and collections, they could also see three-fifths of five, or five-thirds of one, or five-thirds of three (see Figure 2).

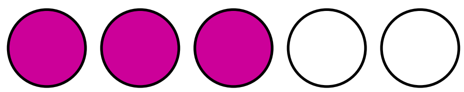

Figure 1. What does this collection illustrate?

They could also see Figure 1 as illustrating that $1 \div 3/5 = 1\,2/3$—that within one whole there is one three-fifths and two-thirds of another three-fifths, or that $5 \div 3 = 1\,2/3$—that within 5 is one 3 and two-thirds of another 3. Finally, they could see Figure 1 as illustrating $5/3 \times 3/5 = 1$—that five-thirds of (three-fifths of 1) is 1. It is an error to think that a particular material or illustration, by itself, presents an idea unequivocally. Mathematics, like beauty, is in the eye of the beholder—and the eye sees what the mind conceives.

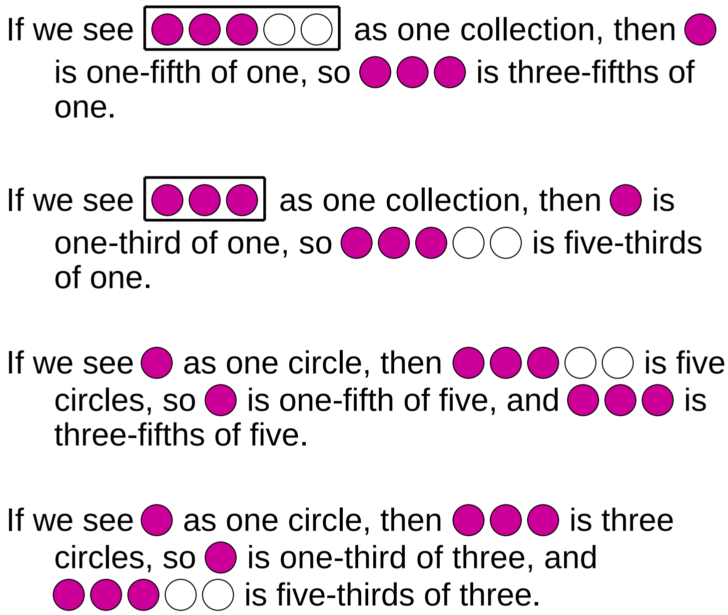

Figure 2. Various ways to think about the circles and collections in Figure 1.

Teachers sometimes understand the discussion of Figure 1 (and Figure 2) as saying that we need to take care that students form *correct* interpretations of materials—namely, the one we intend they have. I actually mean the opposite. It should be our instructional goal that students can make, in principle, *all* interpretations of Figure 1.

A teacher needs to be aware of multiple interpretations of materials in order to hear hints of those which students actually make. Without this awareness it is easy to presume that students see what we intend they see, and communication between teacher and student can break down when students see something other than what we presume.

Also, it is important that *students* can create multiple interpretations of materials. They are empowered when they recognize the multiplicity of viewpoints from which valid interpretations can be made, for they are then alert to chose among them for the most appropriate relative to a current situation. However, it is a teacher's responsibility to cultivate this view. It probably will not happen if the teacher is unaware of multiple interpretations or thinks that the ideas are "there" in the materials.

Figure 1 is customarily offered by texts and by teachers to illustrate $3/5$. Period. In fact, we rarely find texts or teachers discussing the difference between thinking of $3/5$ as "three out of five" and thinking of it as "three one-fifths." How a student understands Figure 1 in relation to the fraction $3/5$ can have tremendous consequences. When students think of fractions as "so many out of so many" they are justified in being puzzled about fractions like $6/5$. How do you take six things out of five?

My second example continues the discussion of fractions. It illustrates that how we think of the materials in a situation can have implications for how we may think about our actions with them.

Suppose, in Figure 3, that the top collection is an example of $3/5$ and the bottom collection is an example of $3/4$. Now, combine the two collections. Does the combined collection provide an example of $3/5 + 3/4$? Yes and no. If we were thinking of 3/5 and 3/4 as ratios (so many out of every so many), then $3/5 + 3/4 = 6/9$ (three out of every five combined with three out of every four gives six out of every nine, as when computing batting averages). If we understood $3/5$ and $3/4$ in Figure 3 as fractions, then it doesn't make sense to talk about combining them. It would be like asking, "If we combine $3/5$ of a large pizza and $3/4$ of a small pizza, then how much of a pizza do we have?" How much of *which kind* of pizza? It only makes sense to combine amounts measured as fractions when both are measured in a common unit. Both answers ($6/9$ and "the question doesn't make sense") are correct—each in regard to a particular way of understanding the concrete material at the outset.

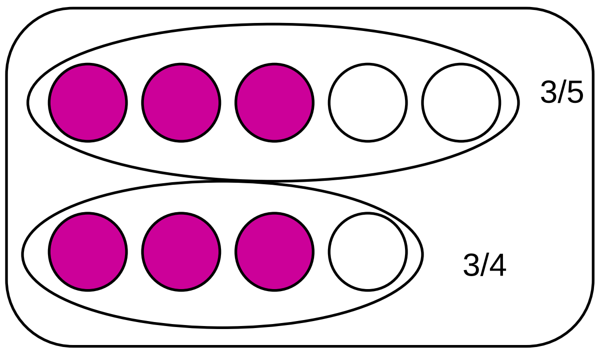

Figure 3. Three-fifths and three-fourths combined into one collection.

Using Concrete Materials in Teaching

It is not easy to use concrete materials well, and it is easy to misuse them. Several studies suggest that concrete materials are likely to be misused when a teacher has in mind that students will learn to perform some prescribed activity with them (Resnick and Omanson 1987; Boyd 1992; Thompson and Thompson 1994). This happens most often when teachers use concrete materials to "model" a symbolic procedure. For example, many teachers and student teachers use base-ten blocks to teach addition and subtraction of whole numbers. Students often want to begin working with the largest blocks, such as adding or subtracting the thousands in two numbers. Teachers often say, "No, start with the smallest blocks, the ones. You have to go from right to left." Would it be incorrect to start with the largest blocks? No, it would be unconventional, but not incorrect.

Since our primary question should be “What do I want my students to *understand*?” instead of “What do I want my students *to do*?” it is problematic when teachers have a prescribed activity in mind—a standard algorithm—and unthinkingly reject creative problem solving when it doesn’t conform to convention or prescription. In cases where students' actions are unconventional, but legitimate, and are rejected by a teacher, students learn once more that “to understand” means to memorize a prescribed activity.

Concrete materials are used appropriately for two purposes. First, they enable you and your students to have grounded conversations. Their use provides something “concrete” about which you and they can talk. The nature of the talk should be how to think about the materials and on the meanings of various actions with them. Such conversations are part of what Hiebert and Wearne (1988) call the *connecting* phase of mathematical learning—constructing strong connections among ways of thinking about concrete situations and conventional mathematical language and notation. Asking the questions “How *do* we think about this? How *can* we think about this? How *shall* we think about this?” is also at the heart of what Thompson, Philipp, Thompson, and Boyd (in press) call “conceptually-oriented” instruction.

Second, concrete materials provide something on which students can act. Our pedagogical goal is that they reflect on their actions in relation to the ideas you have worked to establish and in relation to the constraints of the task as they have conceived it. The discussion of Figure 3 highlighted occasions where you and your students might reflect on the implications of various understandings of fractional amounts for what it means to combine two collections.

Concrete materials can be an effective aid to students' thinking and to your teaching. But effectiveness is contingent upon what you are trying to achieve. To draw maximum benefit from students' use of concrete materials, you must continually situate *your* actions with the question, “What do I want my students to *understand*?”

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