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TECHNOLOGY IN MATHEMATICS EDUCATION RESEARCH: THE FIRST 25 YEARS IN THE *JRME*

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The first 25 years of the *JRME* overlap with the first decades of electronic technologies in education. Hence the growth of technology and research in mathematics education have tended to occur in parallel. But the interactions among mathematics education research, developments in technology, and the evolving nature of school mathematics and learning are complex. To some extent, the technology was superimposed on both school practice and research in mathematics education. On the other hand, it has become increasingly evident that the technology altered the nature of the activity using it.

We will use the metaphor of deep-water ocean waves to illustrate the complex interactions between technology and research in mathematics education. At the surface level are the waves themselves—short term events very much affected by local conditions such as winds and eddies. Then there are swells, of longer duration, of more distant origin, and affected by larger-scale local conditions such as temperature and currents. To distinguish waves from swells requires us to analyze wave behavior over longer periods of time and to situate that behavior within a larger context of interacting forces. Finally, there are tides whose origins are to be found outside the frames of reference for swells and waves and whose behavior is measured in time units an order of magnitude greater than the others. One can focus on any level of wave activity in isolation from the others, describing its behavior and its effects on craft or beaches. But the different levels of activity interact in subtle yet significant ways.

We characterize surface, wave-level studies as those that used calculators or computers as adjuncts to existing curricula and instruction. In such studies the computer or calculator either was used primarily as an aid to computation (Behr & Wheeler, 1981; Creswell & Vaughn, 1979; Gaslin, 1975; Hector & Frandsen, 1981; Hembree & Dessart, 1986; Koop, 1982; Schoen, Friesen, Jarrett, & Urbatsch, 1981; Shumway, White, Wheatley, Reys, Coburn, & Schoen, 1981) or for the delivery of existing content (Fuson & Brinko, 1985; Hativa, 1988; Henderson, Landesman, & Kachuck, 1985; Keats & Hansen, 1972; Kraus, 1981; Kraus, 1982; Robitaille, Sherrill, & Kaufman, 1977). Sometimes, again on methodological grounds, these studies did their post-tests independently of the technology that was used in the intervention. Generally, studies of this type reported weak impacts of the technology on attitude or performance, especially when measured on a delayed basis.

Studies at the "swell" level of change usually involved a closer look at the role of the technology in learning or cognition—how it can be used to support problem solving (Blume & Schoen, 1988; Heid, 1988; Kraus, 1982; Szetela, 1982; Szetela & Super, 1987; Wheatley, 1980) or how it affected students' learning of particular ideas (Ayers, Davis, Dubinsky, & Lewin, 1988; Behr & Wheeler, 1981; Clements & Battista, 1989; Clements & Battista, 1990; Edwards, 1991; Hatfield & Kieren, 1972; Noss, 1987; Olive, 1991; Szetela, 1979; Thompson, 1992; Thompson & Dreyfus, 1988). In these studies the educational activity was more deeply affected by the technology, the researchers were more oriented toward students' mathematical conceptualizations, and they placed less emphasis on controlled comparisons. Research results tended to take the form of analyses of performance and usually led to new questions and new perspectives instead of definitive answers. These studies often used instruction that did not pretend to be an alternative to standard instruction and often focused on in-depth analyses of small numbers of students.

Some studies that simply used the technology to assist in routine computation were not without major paradigm shifts in their goals of instruction, (e.g., Heid, 1988). On the other hand, the deep-seated calculational orientation held by many people in mathematics education (Thompson, Philipp, Thompson, & Boyd, in press) is reflected in many studies' nonconceptual orientation—indeed, about half the studies have a computational skill orientation.

Studies of the effects of programming tended to vary between the wave and swell levels. Some attempted to hold content as fixed as possible, at least as reflected in the measurement of outcomes, for example, algebra in the case of Hatfield & Kieren (1972). However, the underlying activity of writing programs actually offers a radically new experience for the student—such as building a computational model to solve a problem or creating a program that draws novel geometric objects (Clements & Battista, 1989; Clements & Battista, 1990; Noss, 1987; Olive, 1991). These activities were usually not at all present in the existing curriculum. Hence the "content" in terms of the mental activity of the student really cannot be held fixed.

A second class of studies within the swell level, to which we will refer again later, made significant uses of computer environments but as a tacit medium in which the student's mathematical concept formation would be studied (Edwards, 1991; Thompson, 1992; Thompson & Dreyfus, 1988). These studies resemble the Logo studies of Clements and Battista (1989; 1990), Noss (1987), and Olive (1991), and the ISETL (Interactive Set Language) study by Ayers et al. (1988). Here there was no emphasis whatsoever on the computational medium itself or how it worked. The environments were designed to promote the idea that students were to interact with a model that behaved mathematically instead of giving the computer a sequence of programming statements to be enacted.

With 25 years of technology-related research in mathematics education to reflect on but without advance knowledge of the larger picture, we must be tentative in making sense of tide-level changes. Some of these tidal changes involve the shift in research methodology from statistically based comparison studies to cognitive model building based on qualitative studies as discussed elsewhere in this special issue (see Schoenfeld; Steffe & Kieren). Others involve the shift in curriculum content focus, from procedural arithmetic and algebra to problem solving and deeper mathematical reasoning. Accompanying this latter shift is a pedagogical shift to more active and responsible engagement on the part of students.

These tidal-level forces interacted, and continue to interact, with changes in the technology itself. At the simplest level, the content changes took place in part because of the availability of machines that can perform computations that were once done only by hand and some that are entirely infeasible by hand. In addition, electronic media are interactive, which means that a student's actions yield a reaction on the part of the machine, which in turn sets the stage for interpretation, reflection, and further action on the part of the student. Hence we can see an obvious interplay between pedagogical tides that are moving toward increased student control of their learning activities and the technological tide of ever more powerful computational and graphic processing.

SOURCES AND FORMS OF POWER DERIVING FROM NEW INTERACTIVE TECHNOLOGIES

Despite the fact that electronic technologies are reinforcing the pedagogical shift to active learning, it is worth noting that the technology can be used for almost any purpose, including putting students in a passive role or teaching the memorization of facts or rules. Although it carries few intrinsic biases of its own, electronic technology has enormous power to intensify and reinforce almost any bias the user or designer brings to it.

We see three aspects of electronic technologies that enable a deep change in the experience of doing and learning mathematics. One has already been mentioned—interactivity. Until these technologies were available, the media in which one learned or did mathematics were inert: you wrote something and it simply sat there, unchanging. The only interactivity possible involved another human reacting to what you produced. Early uses of interactivity often attempted to mimic the interactivity of humans—for example, the eternally patient tutor or flashcard master, which turned out to be a less than efficient use of computers (Fuson & Brinko, 1985). Indeed, "CAI" initially meant a form of instruction that put computers in the role of a teacher presenting standard skill-based materials. A standing joke in the early

1980s was that a major textbook publisher was looking for a programmer who could put a spiral binding in the middle of the screen.

Over the years, artificial intelligence was vigorously applied to the development of skill-based learning and training systems in the military and elsewhere in education, even in mathematics. Interestingly, the ICAI (intelligent computer-assisted instruction) movement never reached the core of mathematics education research—no ICAI study has been reported in the *JRME*.

A second source of power to change fundamentally the experience of doing and learning mathematics lies in the control available to designers of learning environments. One can engineer constraints and supports, create agents to perform actions for the learner, make powerful resources immediately available to aid thinking or problem solving, provide intelligent feedback or context-sensitive advice, actively link representation systems, control physical processes from the computer, and generally influence students' mathematical experiences more deeply than ever before. Thompson (1992) presented a comparative study that focused on differences between students' work with traditional concrete materials and work with computer-based simulations of those same materials. This study showed the potential for positive impact of embedding constraints and supports directly into the learning environment—as opposed to making them available "off-line" or in teacher instructions and directions. At the same time, this study showed the difficulty of getting students to inject meaning into procedures that they have already automatized meaninglessly.

The third and newest source of power to change the experience of doing, learning, or even teaching mathematics is connectivity. Technologies that link teachers to teachers, students to students, students to teachers, and perhaps most important, that link the world of education to the wider worlds of home and work are only now being developed, and no *JRME* research is reported. Although this is in part due to the novelty of applications of communications technologies, it is also explained by the availability of other venues for the dissemination of such work.

Beyond the tide level is a more subtle, anthropological level of change associated with the technology that is not subject to systematic research. In a certain sense, it is too large and too subtle to measure—it is akin to sea-level changes that might be due to global warming. It involves a gradual reshaping or expansion of human experience—from direct experience in physical space to experience mediated by the computational medium. This shift is well under way in the worlds of work and entertainment. Schools have held it at bay, although they will be unable to do so much longer. The increasingly universal technologically enabled linkages among segments and layers of society will meliorate schools' current isolation. More to the point of mathematics education, practitioners' mathematical activity is increasingly computationally mediated, and ever more mathematics will require a computational medium. Dynamical systems modeling of nonlinear phenomena, for example, simply cannot be done in inert media such as pencil and paper. We know of no systematic research done in the United States on the learning of dynamical systems, although significant activity is developing in this area, e.g., (Devaney, 1992; Sandefur, 1993). Mathematics education research, however, will not likely be a factor in whether dynamical systems modeling, or any modern topic, appears in the mathematics curriculum. Rather, research will, or at least should, inform us of the conceptual linkages among new and old ideas and orientations and how these might be influenced by various instructional strategies and materials. The role of technology, qua technology, will be tacit. As we evolve into cybernetic fish, we will no longer attend to the fact that we are computationally wet.

THE LACK OF TECHNOLOGY-RELATED RESEARCH IN THE *JRME*

We are surprised by how little technology-related research has appeared in the *JRME*. Overall, less than four dozen studies appear, perhaps two-thirds of all issues have no technology-related articles, and entire years have passed without a single article relating to or using electronic technologies—as recently as 1983 and 1984. The 1990s show no change in this pattern. This lack of technology-related work merits some examination. In part it reflects the mathematics education research community's lack of technological engagement, and in part it reflects the development of a technology-oriented research and development community with its own venues for dissemination, such as *Computers in Mathematics and Science Teaching, Interactive Media and Learning*, and *Journal of AI in Education*. Indeed, dozens of publications now attend to technological developments in mathematics and science education, and several professional and scholarly organizations are devoted to these issues. The availability of such non-research-oriented venues suggests (a) that these technologies, although growing in importance and penetration of practice, are not part of the mainstream activity of mathematics education researchers and (b) that they are regarded as the province of specialists in the development and use of these technologies. This development of alternative communities and venues is but one form of the continuing marginalization of technology in mathematics education.

In the early issues of the *JRME*, several technology-based studies appeared, and although the founding editor has an abiding interest in, and enthusiasm for, technology in mathematics education and might have encouraged these, editorial policy or behavior are probably not major factors in the appearance of technology-related research in the *JRME*. The major factor appears to have been and continues to be the paucity of publishable research.

Perhaps the same combination of intellectual and institutional inertia and practical barriers that keeps technology out of schools keeps it out of mathematics education research. To use technology in mathematics education research is intellectually demanding—one must continually rethink pedagogical and curricular motives and contexts. To exploit the real power of the technology is to transgress most of the boundaries of school mathematics practice. And normally, a powerful technology quickly outruns the activity-boundaries of its initial design—students and teachers, as well as its designers, generate activities that were not conceived in the design process. This renders classroom-based research, especially research that extends beyond brief interventions, difficult—and makes direct comparison and tightly controlled experimental studies inappropriate. Thus, virtually all the technology work reported in the *JRME* involves relatively short instructional interventions, usually outside the officially approved curriculum. To use technology in mathematics education research also involves other practical complications, chief among which are the logistical problems of getting subjects and computers in the same location at the same time, and to provide teachers and students with appropriate computer and curricular materials. The computer materials, especially software, are very expensive and time-consuming to produce, and are seldom available off the shelf in the needed form. More generally, we see no hint at research that anticipates the technological circumstances or possibilities of even a few years beyond where we are today. Elsewhere, the first author has discussed the challenge of "proleptic" research (Kaput, 1993)—research that anticipates rather than trails the technological curve.

A revealing example is the widely known work associated with the Geometric Supposers in the mid 1980s, none of which was reported as the subject of research in the *JRME*. Quite clearly, even to a naive observer, the Supposers enabled radical changes in the experience of doing and learning geometry. But research on exactly what changes, and what the cognitive outcomes of this change is and continues to be in very short supply. The more powerful geometry environments of the 1990's are experiencing the same inattention by researchers. Researchability of the larger-scale role or impact of technological innovation is thus an open question.

But the difficulty of doing publishable technology-related research and the development of alternative venues does not entirely explain the paucity of technology-related *JRME* articles. A deeper reluctance to engage technology seems to be at work, a kind of tacit satisfaction with current ways of operating, an acceptance of reaching for what is already accessible, a comfort in today's givens—especially curricular givens. Unanswered questions abound. We don't need new ones, especially ones that are hard and expensive to address. In some ways, the mathematics education community participates in the same conservative attitude as do schools.

With few exceptions, the mathematics education community, and especially researchers, have had a passive attitude towards technology. The latest technological innovation, often a tool created for another audience and set of purposes, is too commonly accepted uncritically, leading to sometimes awkward marriages between learning environments and technological innovation or to retrofitting curriculum and instruction to accommodate the innovation. Calculators built for shopkeepers, shoppers, or engineers often entered an unchanged curriculum; uses of spreadsheets often led to a curriculum that fit the characteristics of a tool built initially for accountants. Only recently have calculator manufacturers sought the advice of educators in the design of their products. And only very recently have computer-based tools been specifically designed for learners. Often, however, such tools are designed by people steeped in the technology but without deep insight into the problems of mathematics education.

But is research necessary to direct reform? As noted earlier, some researchers in the late 1970s began to look at how the availability of calculators affected problem-solving strategies (Behr & Wheeler, 1981; Creswell & Vaughn, 1979; Hector & Frandsen, 1981; Koop, 1982; Shumway et al., 1981; Szetela, 1979; Szetela, 1982; Wheatley, 1980). Of interest is the fact that large-scale curriculum development began to incorporate these innovations a few years later, but it does not appear that this development was research driven—it simply seemed, on the basis of the practical wisdom of the innovators, to be the right thing to do. Indeed, as early as 1980, and with a very narrow sheaf of research to report, the NCTM made a strong statement on behalf of technology use in mathematics classrooms. Recommendation 3 of *An Agenda for Action* states, "Mathematics programs must take full advantage of the power of calculators and computers at all levels" (NCTM, 1980). We may be in a position analogous to what was experienced earlier in this century relative to transportation or communication changes. It was certainly not research that led to the role of automobiles in our society or, more recently, to the role of television. These technologies interacted with other aspects of the society to help transform it, but without planful intervention or control.

Another major arena not addressed within mathematics education, and hence by researchers, is the role of technology apart from direct instruction. Consider, in contrast, the airline industry. It simply could not exist in anything like its current form without a massive information technology infrastructure acting in a support role—scheduling flights, crews, gates, maintenance, routes, ticket prices, reservations, purchases, seat assignments, and so on. In addition, there is a massive use of communication infrastructure and technology that connects the many parts of the system. Contrast this enormously complex system that, despite occasional delays and mishaps, performs remarkably efficiently every day serving millions of passengers, with our educational system and its use of information and communication infrastructures. To expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic.

Will research in mathematics education inform the growth of technology-supported mathematics teaching and learning? Answers to this question will depend on the answers to three others:

1. Will mathematics educators assume responsibility for shaping the roles of new technologies in school mathematics?

2. Will mathematics educators assume responsibility for seeing that uses of new technologies reflect the substance of their curricular and pedagogical ideals?

3. Will researchers turn issues raised by new technologies into researchable questions?

In 1994, a relatively small number of people are involved in technology-related research that goes beyond what could have been done 25 years ago with only slightly different technology. The number of technologically sophisticated graduate students who are being trained to conduct research on the potential of new or emerging technologies is likewise quite small. Thus, the current answer to these three questions is, "Probably not." We hope this situation will change during the next 25 years.

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