MEASURES OF GROUP CHARACTERISTICS

You are familiar with the idea of measuring things -- a person’s height, a steak’s weight, a car’s value (its selling price). The purpose of any measurement is that it allows us to make indirect comparisons. We could compare two people’s heights by having them stand aside each other. By using measurements we can compare two persons’ heights even when they reside on different continents.

In other modules we called the things we measure *quantities*. In statistics quantities are called *variables*. A variable is the property of objects or people on which we wish to collect measurements. The actual measurements are said to be *values* of a variable.

Lengths are probably the measurement most familiar to you. But lengths are atypical of most measurements — we can think of a measured length as if having laid a ruler repeatedly along the thing being measured. But most things we want measured are not so easily conceived. Conceiving the thing to be measured clearly enough to imagine a way to measure it is often the most difficult part of statistics.

Here are other variables that were difficult to measure until the variables themselves were conceived clearly:

- **Stuff-ness**
  How much “stuff” is something made of? This isn’t the same as how much something weighs, for any object weighs less on the moon than it does on earth, even though in each place it is made of the same amount of “stuff”. Scientists addressed this question by creating the concept of *mass*.

- **Square-ness**
  How do you measure two rectangles to determine which one is most square? Is a 3 x 5 rectangle just as square as a 5 x 7 rectangle? [The ratio of two adjacent sides is a standard measure of squareness.]

- **Twist-ness**
Try holding a 10-pound sack of sugar close to your chest. Then hold the same sack of sugar at arms length. It’s harder to do. There is something about the kind of force exerted that makes it greater when we hold it farther out. This same issue is behind the concept of a lever. A small child can balance a heavy adult on a teeter-totter when the adult is close to the center and the child is far out. The central issue is “amount of twist” exerted at a point. [The conventional measure of amount of twist, also called torque, is to multiply the amount of force and the distance the force is being applied from the point around which the twist is made.]

- **Round-ness**
  Some ovals are more round than others. How could we measure the roundness of these ovals to order them from most round to least round?

- **Hard-ness**
  Hardness of objects is measured by ranking them according to what scratches what. Diamond scratches iron, so diamond is harder than iron. Note that a hardness of “8” is not twice as hard as a hardness of “4”.

- **Smart-ness**
  We associate smartness with the ability to solve problems or figure things out. So, we measure smartness customarily by asking people to solve problems. What problems we ask people to solve often is a source of controversy.

- **A person’s preference for foreign cars (versus domestic cars)**

- **Hot-ness (aka temperature)**
  Temperature is a measure of the average kinetic energy of an object’s atoms. Put another way, it is a measure of the object’s atom’s motion. Lots of motion means high temperature. But, we typically measure temperature by examining the height of a column of mercury (or alcohol) in a thin tube. The height of a
column of mercury provides an *indirect measure* of the variable we want to measure actually. The higher the mercury’s temperature, the more active its atoms, and the higher the column’s height.

- **Young children’s parents’ tolerance of violence on television**
  We might measure parents’ tolerance of violence by keeping track of how much violent programming parents knowingly allow their children to watch.

In statistics we often measure something about groups of objects -- like a group of people or a box of apples. But the aim of measuring some characteristic of a group still fits the general aim of measuring anything: *Measurements of group characteristics allow us to compare groups as if they were individuals.*

A common way to measure some characteristic of a group is to add each member’s measurement on that same characteristic.

- **An apple box’s quality:**
  Pick 5 apples at random and add their individual quality ratings.

- **A sales force’s output:**
  Add each salesperson’s output.

- **A basketball team’s performance:**
  Add each team member’s points.

- **A mile relay team**
  Add each team member’s time.

- **A swim team’s performance on a test**
  Add each class member’s score.

- **The quality of a sock collection:**
  Add the number of holes in each sock.

- **A book club’s performance:**
  Add each member’s number of books read.
An auto-shop’s performance:
Total dollar value of jobs completed (sum of each mechanic’s invoices);
Or, the number of jobs completed (sum of each mechanic’s completed jobs).

An elementary school’s attendance for a period of time:
Add each student’s number of days in class during the period of time.

A family’s TV addiction:
Add each family member’s number of hours watching TV.

A nation’s fertility:
Count the number of live births each year.

Value of a cattle herd:
Add each animal’s value;
Determine value by multiplying weight and current selling price.

A farm’s weekly milk production:
Add number of gallons each cow produces in the given week

Corn de-tasseling crew’s rating on a corn field:
Count the number of tassels remaining along the field’s longest diagonal.

Here are some poor examples of group measures. What is problematic about each one?

A flower garden’s life span:
Pick 5 flowering plants and see how long they last.

A kindergarten class’ eye color:
Classify each student’s eye color into one of brown, black, blue, green, hazel, and other.

A car’s fuel efficiency:
Miles driven on a tank of gas.
The quality of television programs:
Pick 5 programs at random and add their individual qualities.

An ice-cream truck’s sales output:
Add each ice cream’s cost.

A newspaper’s delivery status:
Add each paper boy’s deliveries.

A building:
Total number of rooms on each floor.

An elementary school’s fire-drill response:
Add each student’s time to leave the building.

Your turn:
Give 5 more good examples of group characteristics that are measured by adding measures of that same characteristic of the group’s members. Say why your examples are good. Be creative with your examples!!!

An important detail about comparing two groups’ cumulative totals is that it often makes little sense to compare cumulative totals directly when the two groups are different sizes or measurements are taken on different characteristics.¹

These are examples of problematic comparisons — comparisons between groups that do not capture the judgements we hope to make. Say what is problematic about each one.

- Compare a 1600 meter relay team’s total time with a 400 meter relay team’s total time to see which team is faster.
- Compare a 12-member debate team’s total score with a 5-member debate team’s total score to see which team debates better.

¹ But sometimes it does make sense to compare cumulative totals directly even when the groups’ sizes are different. We’ll consider this later.
• Compare a 13-student English class’s total score on Test 1 with an 18-student English class’s total score on Test 1 to see which class scored better on Test 1.

• Compare one apple-box’s quality rating determined with a 10-apple sample with another apple-box’s quality rating determined with a 15-apple sample to see which box contains higher-quality apples.

Here are poor examples of problematic comparisons of group performances. That is, these are examples given by various people to capture problematic comparisons of group performance, but their examples failed to be problematic in a well-defined way. Explain how each one is a poor example of a problematic comparison.

• Compare a 5-member basketball team’s total score compared with a 4-member basketball team’s total score (they played against each other) to see which is the better team. [Hint: When two teams compete head-to-head, the one with the better score wins — regardless of how many players each has. The 4-member team might receive accolades for playing well while short 1 player, but if they score fewer points than their opponent, they lose! The comparison is not problematic with respect to the reason for making it.]

• Compare a group of six-legged dogs’ speeds with a group of four-legged dog’s speeds to see which dogs are faster.

• Compare ten 105-pound persons’ calorie consumption with five 300-pound persons’ calorie consumption to see which group eats more.

• Compare the total cost of a group of 3-karat diamonds with the total cost of a group of 5-karat diamonds to see which is the more valuable group.

• Compare a 50-mango box’s quality score calculated by adding quality scores of 5 randomly selected mangos with a 100-mango box’s quality score calculated by adding quality scores of 5 randomly-selected mangos to see which box has higher quality mangos.
• Compare sales from a tiny corner market with sales from a major supermarket to see which store sells more groceries.

• Compare a $60 pair of shoes at Robinson’s with a $20 pair of shoes at Pay Less Shoes to see which store has better values.

Your turn:
Give 5 more good examples of problematic comparisons between two groups of different sizes. Explain why your examples are good. Be creative!!

A common tactic to sidestep the problem of comparing measurements of some characteristic of different-sized groups is to compare measurements per person or per item. In this way the groups’ sizes don’t matter. We end up measuring a group characteristic by stating the group measurement as if it were a measurement of a member of the group.

A group measurement stated per person or per item is called the group’s average rate. In statistics, a group’s average rate often is called its mean score.

Calculating Average Rates

An average rate is the amount per item which will produce the same cumulative total for the group were each item to contribute the same amount to the group’s score. Calculating a group’s average rate is like sharing the group’s cumulative score evenly among the items composing the group. If a group has \( n \) members, then each member would contribute \( 1/n^{th} \) of the total. So, to calculate the amount each member would contribute -- were they each to contribute \( 1/n^{th} \) of the group’s total -- we divide the group’s total score by the number of members in the group. Thus, the formula for calculating a group’s average rate, or mean score, is
Here are some examples:

A class’s cumulative score on Test 1

Share the total evenly among the class members, as if each student contributed the same score to the total. (Divide the total score by the number of students in the class. The class’s “average score” is the class’s total points per student.)

An apple-box’s quality:

Pick some number of apples at random and add their individual quality ratings. (Divide the total by the number of apples. The box’s “average score” is the box’s total quality per sampled apple.)

A sales force’s output:

Add each salesperson’s output. (Divide the total by the number of sales persons. The company’s “average rate of output” is the company’s total sales per salesperson.)

A basketball team’s performance:

Add each team member’s points. (Divide the total points by the number of players on the team. The team’s “average performance” is the team’s total points per player. Note that this measure isn’t very important, because NBA and NCAA rules state that the winner of a game is the team who scores the most points, not the one who has the highest number of points per player. Team’s might not have the same number of players.)

Your turn:

Give 5 examples where a characteristic of two different-sized groups can be compared directly by comparing their average rates. Design each example so that comparing total amounts will lead to a different conclusion than will comparing average rates.
Comparing Group Measures and Average Rates

Appendix 1 lists 75 cereals and a number of measures taken on them. The second column lists cereals’ manufacturers; the table is sorted by manufacturer.

Examine each column, looking for manufacturers who “stand out” in comparison to other manufacturers. Use Data Scope to make tables like the one below to present your findings.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Variable</th>
<th>Cum Total</th>
<th># Cereals</th>
<th>Ave. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td></td>
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</tr>
<tr>
<td>General Mills</td>
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<tr>
<td>Kellogs</td>
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<tr>
<td>Nabisco</td>
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<td>Post</td>
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<tr>
<td>Quaker Oats</td>
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<tr>
<td>Ralston</td>
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</tr>
</tbody>
</table>

Examine

(Have them compute “Fat Scores”, “Calorie Scores”, etc. for each manufacturer. Then have them compare manufacturers by aggregate score and by average rate (mean score). Discuss how in many cases you would favor one company on aggregate score but favor another when examining average rates.

Perhaps raise the issue of “consistency” of the scores (aka “variability”).