Some Remarks on Conventions and Representations

Patrick W. Thompson
Vanderbilt University

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There is an essential difference between radical constructivism and socioculturism. It is that the former takes social interaction (specifically, communication) as a phenomenon needing explanation, whereas the latter takes it as a constitutive element of human activity. This difference expresses itself most vividly in the types of explanations coming from radical constructivists and socioculturists. The former tend to focus on human discourse as emanating from interactions among self-organizing, autonomous individuals. The latter tend to focus on the collective activity in which individuals participate. That is, from a radical constructivist perspective, what we take as collective activity is constituted by interactions among individuals each having schemes by which they generate their activity and by which they make sense of other's actions. From a sociocultural perspective, collective activity and social interaction are given, predating any individual's participation in it. The individual accommodates to social meaning and practice.

Social interaction, from a radical constructivist perspective, is constituted by individuals' mutual interpretation of what each perceives as other-oriented action. These interactions, collectively and over time, constitute social activity.[i] If we also assume that individuals reflect on their actions, then it follows that each individual's participation changes as she becomes aware of, elaborates, and interiorizes[ii] her activity and her understanding of its repercussions. A radical constructivist perspective on the constitution of collective activity is similar to points of view originating in complexity theory and chaos (Mainzer, 1994; Sandefur, 1993). In complexity theory, the intent is to model complex phenomena by attempting to identify elementary process that, through large numbers of interactions over sufficient amounts of time, regenerate the phenomena. The elementary processes, from a radical constructivist perspective, to account for collective activity are intersubjective operations within individuals and interactions among groups of individuals. This, combined with the facts that people have memories and use them, and that interactions often produce artifacts that people use both informationally and practically, engender social activity from a complexity theory point of view.

The elementary processes of social interaction, from a radical constructivist perspective, are mutual interpretation, mutual accommodation and personal action textured by each. This is similar to symbolic interactionism (Cobb, Jaworski, & Presmeg, 1996; Miller, Katovich, Saxton, & Couch, 1997; Prus, 1996), specifically those forms that treat the fact of communication problematically. From this symbolic interactionist perspective, people do not communicate meanings *per se*. Rather, to say "one person communicates a meaning" to another means that a listener attributed meanings to utterances he perceived, where the production of those meanings are both enabled and constrained by their own understandings and by the image they've built of, or impute to, the speaker (see Figure 1).

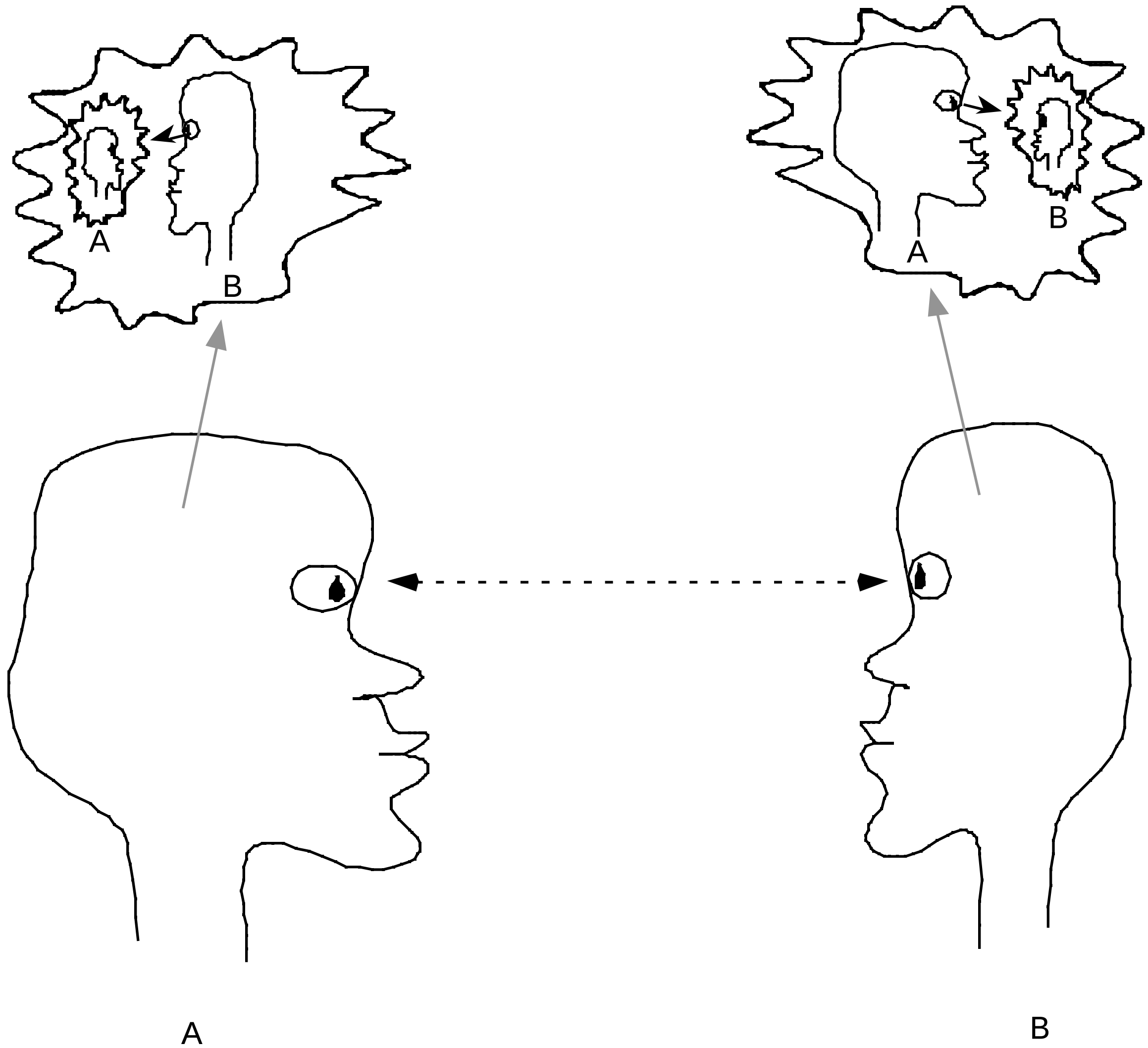


Figure 1. A symbolic interactionist view of one moment in human communication

Figure 1 portrays a symbolic interactionist understanding of two people in communicative interaction, as opposed to physical or asocial interaction. We see the two of them acting, but we understand each to be predicating his actions on an image of the other and upon those action's implications. Each person acts reflexively, and yet acts with consideration of others, much as Baursfeld (1980) described. Figure 1 makes this point a bit too strongly, in that it portrays interactors as thinking this through quite carefully. Instead, I mean that each person imagines and acts, sometimes reflectively but always through imagery built from past interactions. Individuals' actions are textured by current understandings and imagery as much as predicated on them.

As Glasersfeld (1995) notes, to say two people communicate successfully means no more than that they have arrived at a point where their mutual interpretations, each expressed in action interpretable by the other, are compatible – they work for the time being. *Intersubjectivity* is the state where each participant in a socially-ongoing interaction feels assured that others involved in the interaction think pretty much as does he or she. That is, intersubjectivity is *not* a claim of identical thinking. Rather, it is a claim that no one sees a reason to believe others think differently.

The significance of these considerations in regard to issues of representation is twofold:

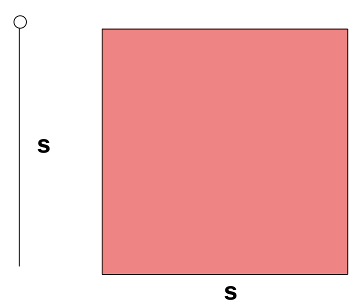
(1) It is clear that, from a constructivist perspective, claims that representations are socially mediated or that representations are mediators of social processes are claims about the surface features of large scale, complex social interactions (as I've described them). Representations, as personal constructs, are creative ways to remind ourselves systematically of ideas we had, connections we made, and operations we applied previously in the presence of operating now. Representations as social conventions are expressions of intersubjectivity (as I've defined it).

(2) Features of large-scale social interactions can be terribly misleading when trying to determjine the nature of individuals' participations in the interactions and meanings that individuals attribute to socially-shared "representations."

An example from a research study (Thompson, 1994)with 4th-year mathematics majors and 1st-year math education graduate students illustrates the second point. The study was on their understanding of rate of change and the Fundamental Theorem of Calculus.

The setting for this example (unreported in the 1994 article) is that we had worked for some time rebuilding their notion of average rate of change. To raise the issues I foresaw as central to making the Fundamental Theorem a discussible idea students needed to realize that constructing a coherent interpretation of formulas and graphs was more problematic than they realized. The class session I describe here lasted for approximately 90 minutes and centered around interpreting a formula that defined an "average rate of change" function in regard to the variation in a square's area expressed as a function of the square's side length (Figure 2). I will describe this session in some detail.

A(s) = 1.633 square inches
s = 1.278 inches



s	A(s)
1.000	1.000
1.111	1.235
1.278	1.633

Figure 2. Square region whose side length and area varies. Table shows various side lengths and associated areas.

A theme of the course up to this session had been to develop a consensual understanding of average rate of change of one quantity (A) with respect to changes in another (B). The "consensual meaning" by this time was "that constant rate of change of A with respect to B which will produce the same change in A as actually happened in conjunction with the actual change in B" (Bowers & Nickerson, In press).

The instructional segment began with students watching the square in Figure 2, displayed in Geometer's Sketchpad, as I increased the length of each side of the square by dragging the endpoint of the vertical segment labeled *s*. After each change in the side length I asked, "What was the average rate of change of the square's area with respect to the change in its side length?" For each computed value (e.g., .235/.111 for the first entry) I asked, "What does it mean that the average rate of change of area with respect to side length was 2.117 in this step?" In this case we proceeded after establishing that it means *if the area were to increase at the constant rate of 2.117 in*^{2} *per inch increase in side length as the square's side length increases from 1 inch to 1.111 inches, then the area would increase precisely as much as it actually did*. We repeated this discussion for several more values of *s* and A(*s*).

We observed that the side of a square, measured in inches, is increasing in length and that the square adjusts appropriately as the side's length increases. The area of the square, in square inches, was given at all times by $A(s) = s^2$. We also established that each value of the function

$$r(s) = \frac{A(s+0.25)-A(s)}{0.25}$$

gave the average rate of change of area with respect to change in side length as the side-length changes from *s* inches to *s*+0.25 inches. This means that for each side-length *s*, if the area were to increase at the constant rate of change given by *r*(*s*) for the next 0.25 inch increase in side length, then the change in the constantly-changing area will be precisely the same as the change in *A*(*s*).[iii]

I displayed the graphs of *A*(*s*) and *r*(*s*) (see Figure 3) and said "Here is a point on the graph of *r* (clicking on a point of *r*'s graph, displaying coordinates (.7667,1.635)). What does this point having coordinates (0.7667,1.635) represent?" I then told students to form groups of two and three and directed each group to arrive at a consensus statement of what is represented by the point (0.7667,1.635). After all were finished, each group reported the result of their discussion. Everyone in every group stated his or her satisfaction that their spokesperson represented their group's interpretation, and everyone stated his or her satisfaction that the groups arrived at essentially identical interpretations. I then asked each person to write his or her answer to the question, "What is represented by the point on *r*'s graph having coordinates (0.7667,1.635)?"

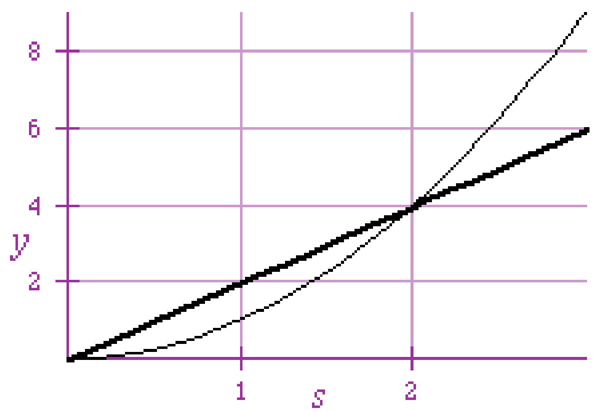


Figure 3. Graphs of A(s) and r(s).

Eleven of 19 students responded to the question. Eight stated that they couldn't remember what their group had said before, that they couldn't reconstruct it, or they couldn't come up with an interpretation. Responses from the other 11 students are given in Table 1.

Table 1. Responses to "What is represented by the point (0.7667,1.635)?"

1. That point represents the average rate of change in the area with respect to the change in length of the side at .7667 inches.

   $$\text{Ave. rate of change} = \frac{1.635 \text{ sq in}}{\Delta \text{ in s in inches}}$$

2. Given side length *s* of .7667 in., the <u>avg rate of change</u> in area with respect to side length is 1.635 sq/in.

3. c=.7667 is the rate change of side length at this point. y=1.635 is the average rate of change to respect of change of *x*.

4. *r*(*x*) represents avg rate of change. Therefore, the specific point represents rate of change from area of sq with side *s* to (*s*+.25) where side L = .7667. Avg. rate of change here is 1.6.

5. x=.7667, y=1.635. At point x = .7667, if the rate of change of *r* were to be constant thereafter, the area would increase by 1.635 every time *x* increased by 1.

6. What does the point x=.7667 and *f*(*x*) = 1.635 mean? The average rate of change in the area in sq in of a square with side .7667 in as it is increased by .25 in.

7. The point represents the area of the square with a side of .7667 using the average rate of change for a change of .25 inches in the side.

8. At *s* = .76 we find the average rate of change between a square of length .76 and a square of length approximately 1.

9. *r*(*s*) represents the rate of change of the area when the side changes from about 0.75 to 1.00.

10. What does the point *s*,*r*(*s*) stand for when *s* = .75? $\frac{r(.75+.25)-r(.75)}{.25}$. $\frac{rateofchangeinarea}{changeins}$.

11. What does $\begin{pmatrix} x \\ y \end{pmatrix}$ represent? An average rate of change of 1.635 units^2 for side length of .7667 units increased by .25 inches.

Two aspects of students' responses are striking. First, none of the responses is internally consistent. Five are relatively close (numbers 2, 6, 8, 9, and 11). Six responses are conceptually incoherent, entailing internally conflicting meanings. Second, no interpretation even remotely resembles those that they spent 50 minutes developing and to which they each expressed satisfaction that they had said what they intended.

The two aspects together, lack of internal coherence in their interpretations and lack of agreement between private and publicly stated interpretations, points to a matter worth considering. When we claim that agreement has been reached on a relatively complex idea because disagreement hasn't been expressed, we must consider the possibility that students haven't analyzed their own or others expressions sufficiently to detect severe inconsistencies.

Therefore, when incorporating an emphasis on “representations” in conceptually-oriented discussions, it may be prudent to exercise caution before concluding that a consensus in meaning or interpretation has been reached. It may also be prudent to be cautious about concluding what individuals understand even when public agreement seems certain. What individuals understand may be expressed as something stable in the way they interact, but the extent to which interaction-as-stable-pattern reflects individuals’ understandings may be uncertain at best.

Notes

[i] These arguments are presented more fully in Steffe & Thompson (in press) and in Thompson (in press)

[ii] I use *interiorize* in a Piagetian sense. For individuals to intereriorize actions means that they construct schemes of operations they may carry out in thought and by which they may anticipate actions’ outcomes.

[iii] I fully appreciate anyone’s right to wonder how valid is my claim that “we established” these interpretations. I employed the criterion that several people offered this interpretation, explained it, and no one objected that the speakers’ interpretation or explanation was problematic.

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