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EPISTEMOLOGICAL ANALYSES OF MATHEMATICAL IDEAS: A RESEARCH METHODOLOGY

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This paper discusses a methodology for researching the question, "What does it mean to understand *x*, and how might people develop such an understanding?". We call this methodology *epistemological analysis* (EA), and we view it as appropriate for creating didactic models of mathematical understanding. We give an overview of aspects of EA by outlining its links to a constructivist epistemology and related research tools and analytic methods.

Introduction

A large part of mathematics education research investigates the question, "What does it mean to understand *x*, and how might people develop such an understanding?" This paper gives an overview of a methodology for researching that question. We call this methodology *epistemological analysis* (EA), and we view it not only as appropriate for creating models of mathematical understanding, but also as supporting the design of instructional strategies intended to support the development of such understanding.

The aim of EA in any setting is to produce an *epistemic subject*, a framework for creating models of individual persons' mathematical thinking and knowing that explain why they behave as they do, both individually and in interaction with others. We borrow this phrase from Piaget, who described the epistemic subject as "the cognitive core that is common to all subjects at the same level" (Piaget, 1970, p. 120, Quoted in Glasersfeld, 1995, p. 72). However, we use this term slightly differently than did Piaget. To Piaget, something like "the conserving child," the child who conserves one-one correspondences even after spatial transformations, would have been an example of an epistemic subject. It is a description of a "common" child at a particular stage of development. We extend the notion of epistemic subject to reflect Piaget's aim to develop theoretical frameworks that describe knowledge in a particular area as being organizations of mental operations any of which could be at one of several levels of development within any particular person. When accomplished, the framework turns out to be useful in describing the actual composition of any one person's knowledge when positing where in a developmental sequence each mental operation is. When the framework is particularized for a particular person or group of persons, you have a model of that person's, or group of persons', ways and means of operating within the contexts being investigated.

An overview of epistemological analysis

Characterizations of an epistemic subject must have very special natures because of the traditions from which EA draws its notions of knowledge and knowing (radical constructivism and thereby operational analysis and genetic epistemology) and because of the tools it employs to research them (cognitive task analysis, constructivist teaching experiment, and conceptual analysis). Developing these links completely is beyond the space limitations of this paper. Instead, we offer a brief overview of EA's intellectual heritage as a way to describe its aspects.

Task analysis (TA) focused on performance capabilities a person must have in order to perform a specific mathematical task (Gagné, 1977; Thorndike, 1922). Where TA focused on behavior, cognitive TA viewed behavior as an expression of goal-directed cognitive operations (Klahr & Siegler, 1978; Newell & Simon, 1972). Cognitive TA became very influential in some areas of mathematics education research (Brown & VanLehn, 1981; Davis, Jockusch, & McKnight, 1978; Greeno, 1978; Resnick, 1975; Schoenfeld, 1989). It provided not only models of cognitive processing, but a method for describing the knowledge that schooling should help students develop (Bransford, Nitsch, & Franks, 1977). One shortcoming of TA, both behavioral and cognitive, was that people using it often confounded correct performance and understanding, thereby ignoring issues of individuals' motives and conceptions of the contexts in which their behavior was observed (Cobb, 1987; Steier, 1991). One positive aspect of cognitive TA was that it produced methods for modeling complex organizations of cognitive processes and it developed criteria for assessing a model's viability — would a model "run" if operationalized and would the model be expressed in behavior roughly consistent with observations?[i]

Piaget's *genetic epistemology* (Piaget, 1971, 1977) was an interdisciplinary approach to understanding human intellectual, moral, and social development. Genetic epistemology made deep connections among biology, philosophy, psychology, and logic, and used both structural and functional approaches to understanding what might constitute human knowledge. The ideas that knowing is always a dynamic process, always involving mental operations, and that mental operations are always part of a larger system of operating, were central to Piaget's work. On the other hand, while Piaget described mental structures as being organizations of mental operations, he emphasized the structural aspect of knowledge over the operational aspect of knowing. But he always grounded his notion of knowledge firmly in the idea that knowledge is not a copy of reality, but rather is built from and within a person's total neural activity. Working from a tradition distinctly different from American psychology and independently of the Piagetian school, Silvio Ceccato outlined what he called *tecnica operativa*, or operational technique, in which one must "consider any mental content (percepts, images, concepts, thoughts, words, etc.) as a result of operations" (Cecatto, 1947 as cited in Bettoni, 1998). That is, one must describe *consapevolezza operativa*, or conceptual operations (translated literally as "operating knowledge")[ii] in order to answer the question "which mental operations do we perform in order to conceive a situation in the way we conceive it?" (Bettoni, 1998).

Glasersfeld combined aspects of Ceccato's operational analysis and Piaget's genetic epistemology to devise a way to talk about reasoning and communicating as imagistic processes and of knowledge as an emergent aspect of them (Glasersfeld, 1978). This produced an analytic method, that he called *conceptual analysis* (CA), whose aim was to describe conceptual operations that, were people to have them, might result in them thinking the way they evidently do. CA resembles TA in its attention to detail and its focus on creating models of thinking and reasoning that ostensibly explain why people behave as they do, both individually and in interaction with others, but it differs from TA in the nature of the operations it posits. Where TA describes reasoning as production systems, systems of "if-then" propositions, CA describes reasoning as grounded in imagery and meaning, where meaning and imagery are described in terms of conceptual operations someone might employ to have them. CA's emphasis on conceptual operations derived from Ceccato's operational analysis. Glasersfeld's key contribution was to tie Ceccato's operational analysis to Piaget's genetic epistemology. The combination produced a way of talking about a person's conceptions of specific situations with an eye toward placing them in a context of larger systems of knowing.

In the same way that Glasersfeld accommodated operational analysis to the constraints of genetic epistemology's conception of knowledge, a number of researchers accommodated the Soviet-style teaching experiment, with its roots in Vygotsky's socioculturalism, to conceptual analysis and genetic epistemology (Steffe, 1991; Steffe & Thompson, 2000b; Thompson, 1979). The constructivist teaching experiment's primary purpose has always been to have second-order models of students' understandings built by observers who are reflectively aware of interactions with them. In a sense, the constructivist teaching experiment produces the interactions between students and knowledgeable persons that are at the root of Vygotsky's notion of cultural transmission (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997) and allows investigations of the emergence of intersubjectivity (Steffe & Thompson, 2000a) among participants in instructional settings.

EA is not a new methodology as much as it is a synthesis of those already described. Its purpose is to create models of knowing that attempt to characterize students' understandings in specific settings. At the same time EA also allows us to place those understandings within large systems of knowing in a way that can be useful across students who differ greatly in their conceptions and capabilities. EA's greatest overlap in analytic method is with Glasersfeld's CA and it owes its greatest intellectual debt thereto, but there are differences between them. First, EA is used to model what might be called systems of ideas, like systems of ideas composing concepts of numeration systems, functions and rate of change, or even larger systems like those expressed in quantitative reasoning. The added complexity of modeling large knowledge systems leads to issues unaddressed by CA, issues such as what might constitute principled, coherent, and general conceptions of sophisticated ideas and how immature conceptions of sophisticated ideas might evolve into them. Second, by its use of constructivist teaching experiments, epistemological analysis gives explicit attention to instructional contexts in students' construction of these conceptual systems. These two points support our assertion that models generated by EA have potential didactic value; because they are created in tandem with the design of instructional environments and activities intended to support the development of students' ways of reasoning and thinking, these models capture aspects of how various levels of understanding might evolve into sophisticated, advanced, coherent understanding as a function of students' engagement in instructional activities. They therefore necessarily address the nature of instructional strategies someone might take to foster that development.

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[i] A profound difference between EA and TA that we cannot develop here is that TA is rooted in encodingism while EA is not (Bickhard, 1991a, 1991b).

[ii] Quotations translated by M. Bettoni. Phrases translated by GO Translations, http://translator.go.com./.

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