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INVESTIGATING THE RELATIONSHIP BETWEEN MATHEMATICAL UNDERSTANDING AND TEACHING MATHEMATICS

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In work with pre-service teachers, it became evident that "knowledge of mathematics for teaching," though valuable, was not "fine-grained" enough to be of use. In this short paper, we will discuss our work that examines what it means for pre-service mathematics teachers to develop mathematics content knowledge for conceptual teaching. Our aim in this investigation was to understand the influence of pre-service teachers' [PST] particular understandings of mathematics content (as developed by in a university course setting) on their school-based teaching practices. In doing this, we focused first on PSTs' understandings of mathematics as the primary resource upon which they draw while teaching. The importance of teachers' knowledge of content has been acknowledged by a variety of scholars (Ball, 1993; Grossman, Wilson, & Shulman, 1989; Schifter, 1990). However, it is axiomatic that a teacher's knowledge of mathematics alone is insufficient to support his or her attempts to teach for understanding. In that vein, Shulman (1986) coined the phrase pedagogical content knowledge [PCK]. Ma (1999) and Stigler and Hiebert (Stigler & Hiebert, 1999)further refined the idea of PCK by arguing that teachers need a profound understanding of mathematics – knowledge having the characteristics of breadth, depth, and thoroughness.

In previous work with student teachers (Silverman, 2004) we noted that PSTs naïve conceptions of "profound" understandings of mathematics are inconsistent with teaching mathematics for understanding. Since teachers' understandings of mathematics enable or constrain their ability to orchestrate mathematical discussions that provide students with opportunities to make sense of advanced mathematical ideas, it is important for teacher educators to understand both the understandings with which PSTs enter our programs and ways in which those understandings can be productively influenced. By teachers' understandings of mathematics we mean "the loose ensembles of actions, operations, and ways of thinking that come to mind unawarely – of what they wish their students to learn, and the language in which they have captured those images" (Thompson & Thompson, 1996, p. 16). It is against the background of the images that teachers hold with regard to their own understandings and of the understandings they hope students will have that they select tasks, pose questions, and make other pedagogical decisions

In our recent work with pre-service teachers, we studied a group of PSTs who took part in a course designed to position the PSTs to develop a more advanced, connected understanding of the concept of function (in this case, the more advanced conception is function as covariation of quantities – a conception of function that is consistent with the calls of the NCTM *Standards*), and their subsequent interactions with high school students. Despite the fact that the pre-service teachers did develop a more robust, coherent understanding of functions as covariation of quantities – an understanding of function that supported their ability to speak conceptually about functional relationships – their instruction remained grounded in a variant of the more traditional correspondence conception of function. Analysis allowed us to characterize the pedagogical conceptions of functions that grounded the PSTs plans for instruction and work with the high school students. The conception of coordinate systems, functions and covariation that grounded

the course instruction (Covariational Conception) and the PSTs' pedagogical conceptions are shown below in Table 1.

Covariational Conception	PSTs' Pedagogical Conceptions
Coordinate Systems are used to locate points, which are values of the variables that occur simultaneously	**Coordinate Systems** are where you plot points, which tell you information about corresponding values of the variables. The corresponding values are found by using rule for determining the specific coordinates. Once students' know the rule, they need to practice locating the points until they get comfortable with it.
A **Function** is a relationship between two variable quantities	**Functions** are what we use to find the corresponding values of the variables. Sine and cosine are periodic functions, which means that the graph repeats itself. It is important for the high school students to know are values of sine and cosine of 30-60-90 and 45-45-90 right triangles so that they can see the values.
Covariation involves making sense of the way that two (or more) variables vary together. This involves considering how a dependent variable(s) vary over intervals of an independent variable	**Covariation** is everything that goes on between the points that you plot.

Table 1: PSTs Pedagogical Conceptualizations

Though space to provide details of the data and analysis is limited, these results are indicative of the study-at-large, which indicates that rather than teaching the way one was taught, one teaches what they know – an individual's understandings of mathematical content and their pedagogical conceptualizations of the content are the lens through which instructional activities with students are conceptualized. Thus, this research indicates that professional development efforts must be grounded in helping the PSTs develop particularly powerful pedagogical conceptualizations of the mathematics that they are to teach. It is only then that the PSTs mathematical and pedagogical understandings can support the orchestration of reflective conversations designed to position students to come to develop true mathematical understanding.

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