AFFECTING TEACHERS' LEARNING COMMUNITIES BY AFFECTING THEIR MATHEMATICAL KNOWLEDGE[1]

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Many studies and scholarly articles state the importance of professional learning communities to sustain improvements in mathematics teachers' instructional practices. // Few articles question how teachers' mathematical understandings affect the nature of the professional learning community in which they participate. // In this paper we describe preliminary results from a research project, *Teachers Promoting Change Collaboratively* (TPC[2]), that investigates the effect of a specially-designed course on the ways their learning communities develop. // The course is designed to help teachers develop deep, coherent understanding of the mathematics they teach and to understand what it means to understand this mathematics. // We hypothesized that learning communities composed of teachers who are involved in the course would take profound understanding of mathematical ideas as their core commitment to a greater extent than were they not involved in such a course. //

The study involved 32 high school teachers in seven groups composed of 3 to 6 teachers each. Each group was composed of teachers at a single school. Project leaders recruited groups by giving presentations at area high schools. Groups were not selected randomly. //

Each group had a project facilitator to help teachers problematize their instruction and to assist in organizing discussions. // Groups met weekly in *Reflecting on Practice Sessions* (RPSs) to discuss matters of instructional practice, curriculum, and student learning. Project leaders designed agendas for the RPSs to include specific activities, such as interviewing students and designing a lesson for conceptual understanding. All RPSs were videotaped for later analysis. // Four groups were involved in the course and three were not (Table 1)[2]. All RPS agendas were designed so that they did not rely on issues or activities that were specific to the course. //

Table 1: Teacher Groups in TPC[2]			
Not in Course	Group A	Group C	Group D
	5	5	6
In Course	Group E	Group F	Group G	Group H
	3	4	4	5

The Course

The course met once per week for 15 weeks. It comprised five interleaved phases: (1) Visions of mathematics teaching, (2) introduction to covariation; (3) trigonometric functions; (4) covariation and modeling; (5) action and process conceptions of functions. // In our usage, "covariation" means to track the values of two quantities as they change simultaneously. // We used "function" to mean an invariant relationship between two covarying quantities. We used "model" to mean a representation of quantitative relationships in a situation. // A model often involves functions when the modeled situation is dynamic, meaning that the situation could be thought of as involving quantities that vary simultaneously and which have invariant relationships among them. //

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[2] Groups A and B were at the same school. Three teachers left the project, so Groups A and B were consolidated into Group A.

The issue of coherence was central to the course, in the sense that we always insisted that teachers strive to make sense of ideas so that they "fit" with other ideas. // For example, we insisted that any meaning of angle measure be one that supported standard treatments of the trigonometry of triangles as well as supporting the idea of a trigonometric function having a real-valued argument. // We also insisted that the concept of angle measure support the idea of composition of trigonometric functions[3], and that a trig function's argument and value be measurable in the same unit. For example, we insisted that teachers be able to answer questions and explain their answers, to questions like, "What is the value of cos(30°), in degrees?" // The idea of angles being measured by arc length of a subtended circle measured in any unit that is proportional to the circle's radius satisfies all these requirements. //

Results

Both non-course groups and course groups improved on measures of community formation. However, there were large differences between the two as detected in their RPS interactions and in their beliefs about what the RPSs provided them. // Specifically, at the end of the term, course teachers (i.e., teachers in the course):

- gave lower value to receiving classroom-ready instructional materials

- showed more interest in reflecting on the content they teach

- gave higher value to the activity of lesson study

- gave higher value to issues of mathematical meaning and coherence

- gave higher value to finding out about their students' thinking as a result of instruction

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[3] For example, $\cos(x)$ must produce a value that can be seen as the measure of an angle, for otherwise $\cos(\sin x)$ has no meaning.

- were more likely to have redesigned parts of their curriculum on their own initiative (i.e., not part of any RPS activities) //

Discussion

Our results suggest that a course that focuses on meaning and coherence of mathematical ideas that teachers teach can be a valuable addition to professional development programs that employ learning communities as a medium for teachers' individual improvement. // Our conclusion is limited by the small number of teachers involved and by the lack of random assignment. Nevertheless, the differences were striking, which suggests that this approach is worthy of further study on a larger scale.

Our project continued after this study with the same teachers. They are currently involved in a semester-long lesson study project. // Course groups are designing, implementing, and evaluating a five-day unit on trigonometry. Non-course groups are designing, implementing, and evaluating five-day units on topics of their own choice. // The striking difference seen in the prior study between non-course and course groups is now even more striking based on the quality of the products and on the processes teachers engaged in to produce them. // We will analyze these data after the semester ends with the hope of gaining insight into sources of differences between groups in their beliefs, values, and commitment to mathematical integrity.