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Mathematical Meanings of Korean and USA Mathematics Teachers for Mathematical Ideas They Teach[1]

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There are many studies of teachers' mathematical knowledge for teaching mathematics, mostly at pre-high school levels. In my presentation I will give an overview of the MMTsm and comparative results from the USA and Korea. In this brief summary I discuss one item as an example of items on the MMTsm, and present comparative results on it. The item focuses on one aspect of teachers' meanings for function notation—the use of letters to represent the function's argument. Additional items in the MMTsm on function notation suggest that Korean teachers' meanings for function notation are more nuanced than this one item reveals. My conference presentation will address general results and comparisons, as well as comparisons between Korean high school and middle school teachers.

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There are many studies of teachers' mathematical knowledge for teaching mathematics, mostly at pre-high school levels. A growing number of articles address high school teachers' and university instructors' MKT (e.g., Boston, 2013; Herbst & Kosko, 2014; Lai & Weber, 2014; Lewis & Blunk, 2012; McCrory, Floden et al., 2012; Melville, Bartley, & Fazio, 2013; Seung, 2013; Steele, Hillen, & Smith, 2013), but the vast majority of studies are at the elementary level.

A major criticism of research on teachers' MKT is that *knowledge*, the central construct of MKT, is rarely defined and is therefore operationalized inconsistently across investigations (Thompson, 2013, 2015). Moreover, *knowledge* is often used to mean declarative knowledge,

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which separates knowers from what is known. Mason and Spence (1999) introduced the distinctions among *knowing-to* (act in the moment), *knowing-that*, *knowing-how*, and *knowing-why* (an action is appropriate), arguing that *knowing-to* and *knowing-why* are the most important forms of knowing for teachers. In (Thompson, 2013, 2015) I argued that a focus on teachers' meanings is in line with Mason and Spence's argument, and that a focus on teachers' mathematical meanings is more profitable for understanding their instructional decisions, both in planning and in moments of teaching. I also argued that a focus on teachers' mathematical meanings positions us to help them improve their instruction.

I and my research team developed a diagnostic assessment of teachers' mathematical meanings that we call *Mathematical Meanings for Teaching secondary mathematics* ((MMTsm; Thompson, 2015). It is a 46-item instrument that gives teachers' opportunities to express their mathematical meanings in regard to variation and covariation, functions (notation, models, and properties), proportionality, rate of change, frames of reference, magnitudes, and structure. We translated 43 items of the MMTsm to Korean using standard translation-retranslation methods and, in collaboration with Oh Nam Kwon's research group, administered it to 364 Korean mathematics teachers (263 high school, 101 middle school; **Table 1**). Hyunkyoung Yoon trained a five-person team of Korean mathematics education Ph.D. students on scoring teachers' responses, conducting inter-scorer reliability (ISR) tests during training, during scoring, and after scoring to ensure consistency among scores. We administered the MMTsm to 250 US high school teachers using the same methods (**Table 2**).

Table 1. Korean teachers, school level by major

	Math	**MathEd**	**Other**	**total**
K HS	81	175	7	263
K MS	33	49	19	101
total	114	224	26	364

Table 2. High school teachers by nation and major.

	Math	**MathEd**	**Other**	**total**
Korea	81	175	7	263
USA	63	81	106	250
total	144	256	113	513

In both tables, "Math" means that a teacher reported having a degree in mathematics, either bachelors or masters, whereas "MathEd" means that a teacher reported having a degree in mathematics education. "Other" means that a teacher reported a degree that was neither

Math nor Mathed. We made these conversions when a teacher reported a bachelor degree in one area and a master degree in another: Math/*→Math; */Math→Math; MathEd/Other→MathEd; Other/MathEd→MathEd.

In my presentation I will give an overview of the MMTsm and comparative results from the USA and Korea. In this brief summary I discuss one item (*Figure* 1) as an example of items on the MMTsm, and present comparative results on it. The item focuses on one aspect of teachers' meanings for function notation—the use of letters to represent the function's argument.

Here are two function definitions.

$$w(t) = \sin(t-1) \text{ if } t \geq 1$$

$$q(s) = \sqrt{s^2 - s^3} \text{ if } 0 \leq s < 1$$

Here is a third function *c*, defined in two parts, whose definition refers to *w* and *q*. Place the correct letter in each blank so that the function *c* is properly defined.

$$c(v) = \begin{cases} q(\_\_\_) \text{ if } 0 \leq \_\_\_ < 1 \\ w(\_\_\_) \text{ if } \_\_\_ \geq 1 \end{cases}$$

두 함수가 다음과 같이 정의되어 있다.

$$w(t) = \sin(t-1) \text{ if } t \geq 1$$

$$q(s) = \sqrt{s^2 - s^3} \text{ if } 0 \leq s < 1$$

세 번째 함수 c 는 w 와 q 두 부분으로 정의되어 있다.
함수 c 가 알맞게 정의되도록 빈 칸을 채우시오.

$$c(v) = \begin{cases} q(\_\_\_) \text{ if } 0 \leq \_\_\_ < 1 \\ w(\_\_\_) \text{ if } \_\_\_ \geq 1 \end{cases}$$

Figure 1. Function item (in English and Korean) that is intended to elicit one aspect of teachers' meanings for function notation. © 2015 Arizona Board of Regents. Used with permission.

The item in *Figure* 1 gives teachers the opportunity to apply the aspect of their meanings for function notation having to do with the use of letters. As with other items in the MMTsm, we scored teachers' responses according to a rubric that we developed by examining teachers' responses in the MMTsm's development phase, both to the item as administered as part of the MMTsm and responses to the item in individual interviews.

Table 3 presents the scoring rubric for *Figure* 1. The intent of Level 3 is the teacher responds consistently with a scheme for function notation that the letter used on the left is a placeholder for the function's argument, and is the letter that should be used on the right to specify what should be done with the function's argument to produce a value. The letter's use is entirely local to the function's definition. It is a placeholder for any argument that might be passed to the function for evaluation. Levels 2 and 1 reflect a teacher's orientation to see the letter used in a function's definition as an essential part of the function's name, that this letter should be used any time the function is evoked.

Table 3. Scoring rubric for item in Figure 1.

Level 3:	The response matches the following exactly: $c(v)=\begin{cases} q(\underline{v}) & \text{if } 0\le \underline{v}<1 \\ w(\underline{v}) & \text{if } \underline{v}\ge 1 \end{cases}$ and does not reassign variables, as in "Let *v* = *s*" or "*v* = *t*."
Level 2:	The response matches one of the following exactly: $c(v)=\begin{cases} q(\underline{s}) & \text{if } 0\le \underline{v}<1 \\ w(\underline{t}) & \text{if } \underline{v}\ge 1 \end{cases}$ $c(v)=\begin{cases} q(\underline{t}) & \text{if } 0\le \underline{v}<1 \\ w(\underline{s}) & \text{if } \underline{v}\ge 1 \end{cases}$ $c(v)=\begin{cases} q(\underline{v}) & \text{if } 0\le \underline{s}<1 \\ w(\underline{v}) & \text{if } \underline{t}\ge 1 \end{cases}$ $c(v)=\begin{cases} q(\underline{v}) & \text{if } 0\le \underline{t}<1 \\ w(\underline{v}) & \text{if } \underline{s}\ge 1 \end{cases}$ and does not reassign variables, as in "Let *v* = *s*" or "*v* = *t*."
Level 1:	The response matches one of the following exactly: $c(v)=\begin{cases} q(\underline{s}) & \text{if } 0\le \underline{s}<1 \\ w(\underline{t}) & \text{if } \underline{t}\ge 1 \end{cases}$ or $c(v)=\begin{cases} q(\underline{t}) & \text{if } 0\le \underline{t}<1 \\ w(\underline{s}) & \text{if } \underline{s}\ge 1 \end{cases}$ and does not reassign variables, as in "Let *v* = *s*" or "*v* = *t*."
Level 0:	***Any*** of the following: – The response does not fit a higher level. – The scorer cannot interpret the response. – The response consists of scratch work with no clear indication of a final answer. – The response does not address the prompt; that is, the response is off-topic (see Purpose and Rationale). – The page contains no work, but does contain at least one mark to suggest that the teacher saw this item.
IDK:	The response consists only of "I don't know", or something equivalent that suggests that the teacher is unsure of how to respond. If the teacher stated uncertainty and gave an additional response, score the response ignoring the uncertainty.
X:	The page is completely blank.

Table 4 reports results for Korean high school teachers; **Table 5** reports results for US high school teachers. **Table 4** shows a large majority of Korean high school teachers re-

sponding at a level that suggests their meaning for function notation includes a distinction between a letter used in the definition of a function and a letter used to represent a value that will be passed to the function's definition for evaluation. **Table 5** shows a different pattern of responses among US high school teachers. It shows a much greater tendency among US high school teachers to think of the letter in a function's definition as being the letter you must use whenever a reference to that function occurs. We call this tendency "Function Notation as Idiom", meaning that teachers view the inscription "f(x)" as being, in essence, a four-character "y".

Table 4. Korean HS teachers' responses to item in *Figure* 1. Cell contents: Count (percent of row total)

Major/Level	0	1	2	3	IDK	X	total
Math	18 (22.2)	5 (6.2)	0 (0)	56 (69.1)	0 (0)	2 (2.5)	81 (100)
MathEd	20 (11.4)	7 (4)	1 (0.6)	142 (81.1)	2 (1.1)	3 (1.7)	175 (100)
Other	1 (14.3)	2 (28.6)	0 (0)	4 (57.1)	0 (0)	0 (0)	7 (100)
total	39 (14.8)	14 (5.3)	1 (0.4)	202 (76.8)	2 (0.8)	5 (1.9)	263 (100)

Table 5. US HS teachers' responses to Item 1. Cell contents: Count (percent of row total)

Major/Level	0	1	2	3	IDK	X	total
Math	6 (9.5)	24 (38.1)	5 (7.9)	24 (38.1)	1 (1.6)	3 (4.8)	63 (100)
MathEd	8 (9.9)	28 (34.6)	3 (3.7)	34 (42)	6 (7.4)	2 (2.5)	81 (100)
Other	13 (12.3)	46 (43.4)	4 (3.8)	26 (24.5)	8 (7.6)	9 (8.5)	106 (100)
total	27 (10.8)	98 (39.2)	12 (4.8)	84 (33.6)	15 (6)	14 (5.6)	250 (100)

Additional items in the MMTsm on function notation suggest that Korean teachers' meanings for function notation are more nuanced than this one item reveals. My conference presentation will address general results and comparisons, as well as comparisons between Korean high school and middle school teachers.

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