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Getting Ahead: With Theories*

I Have a Theory About This

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> "Don't paint the material of the sleeve. Become the arm! Get your love into it." (Newell Wyeth to his daughter, Carolyn. In [Meryman, 1991, p. 100].)

Andy has developed a formidable challenge. He wishes mathematics and science educators to develop a predilection toward theory and theory building. To understand what Andy means by this is not simple. I suspect that the more familiar you become with his work the deeper will be your appreciation of what Andy has in mind. I encourage all to become concretely intimate with Andy's point of view. There is much to be gained.

When Andy spoke of theories he referred to theories in the social sciences. I feel uncomfortable speaking so generally, so I will confine myself to theories of learning mathematics.[1] This is not overly restrictive if we take broad views of learning and of mathematics. To learn mathematics is to learn ways of reasoning, so we automatically include mathematical reasoning. Children do not learn mathematics in isolation of a social context, so automatically we include teachers and teaching. Teachers learn (and often re-learn) the mathematics they teach, so automatically we include teachers' learning. Explication is part of mathematical reasoning, so automatically we include communication, and thus we include teaching. This is the context in which I frame my remarks.

I will address three questions in discussing Andy's paper: 1) What is theory for? 2) What is theory about? and 3) When is theory useful? In many respects these questions cut across the issues Andy raised instead of building on them. My defense is that I hope addressing them increases the dimension of the discourse instead of being irrelevant to it.

I want to make clear that my first paragraph is more than laudatory. It opens a

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[1] From here on I will say "mathematics" when I in fact I mean "mathematics and science".

theme I want to develop. Andy's ideas about theorizing in mathematics and physics education stem from his strong, personal image of doing mathematics and physics creatively. On one hand this is hardly surprising. *Anyone's* theorizing about understanding stems from their images of what understanding is like. On the other hand, Andy's thinking about the goals of theory are textured by his highly principled knowledge of mathematics and physics.[2] I admit that technical knowledge of a subject matter is insufficient to guarantee insight into matters of understanding. But I do not hesitate to claim that studying a subject deeply and conceptually[3] provides an experiential basis for studying what it means to understand. If Andy's wish is realized, I suspect that the theorizing he envisions will be done by people who have built deep conceptualizations of the subject matter of which the theories pertain.

Finally, I will follow one of Andy's heuristics: Take a line and push it until it breaks. I will state my thinking about theory and theory building forcibly and await the crash of hammers.

What is theory for?

We are in the business of improving people's learning of mathematics. We focus sometimes on the learner, sometimes on the teacher, and sometimes on both. But our ultimate aim is to improve learning. This is the activity from which we draw our problems. It is an article of faith that insightful solutions to problems begin with understanding the problem. Principled understandings are the most productive, for they allow us to solve problems larger than the one we faced. We become theoreticians once we orient ourselves to developing principled understandings of learning and understanding.

Here I make my radical constructivism explicit. When we theorize about mathematics learning and understanding, our theories must aim to account for mathematics learning and understanding—*including our own (mine and yours, whether pedagogue, researcher, or practicing mathematician)*. If they apply only to children, then the mathematics of our theories is impoverished, and is probably the mathematics of schools (at least as they exist now). Skemp (1979) made the observation that his model of intelligence was more powerful than Skinner's behaviorism, for it had the potential to account for Skinner's and Skemp's activities as theoreticians, whereas Skinner's

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[2] I especially encourage you to read Abelson and diSessa (1981). Here you will not see theorizing on mathematics or physics learning. Rather, you will gain insights into Andy's image of *doing* mathematics and *doing* physics.

[3] Here I must remain vague. By "studying a subject conceptually" I mean at least that one comes to envision techniques, conventions, and methods in relation to goals and motivations.

behaviorism did not. Children grow up. They become adults. They become us. We are *never* blank slates, and our theories must be sensitive to this. Here I address the mathematics education community. Our school mathematics curriculum is conceptually incoherent, and so is mathematics instruction in the majority of school classrooms. A minority of students do learn something of value, but it is not because of any systematicity in the curriculum. A practical aim of our theory-building is to re-conceptualize the curriculum so that it is at least conceivable that someone can learn it. To re-conceptualize the curriculum, however, we need to have principled understandings of the learning we wish to happen in the children experiencing it.

Andy doesn't say so, but in reading his work it seems evident that he operates under the constraint that adult science must be explainable as an outgrowth of children's science. He operates under a strong constraint of coherence in his theorizing about learning physics. We must also operate under the constraint that our mini-theories (to use Andy's phrase) of learning mathematics be coherent with each other and with what we personally understand about mathematics. If we make this coherence operative in our theorizing, we might make disconfirmable theories.

What is theory about?

Andy alluded to Alan Newell's article "You can't play 20 questions with nature and win" (Newell, 1973b) when speaking of the necessity of theories. In that same year Newell published an article on distinctions between process and structure (Newell, 1973a), noting that whether we consider something to be process or structure depends on our grain of analysis.[4] In this same regard, the texture of our theories of mathematics learning will be dependent upon our grain of analysis. Our grain of analysis will be influenced heavily by two considerations: the learning we wish to explain and the community with which we wish to communicate. Learning as a neurological phenomenon is at one extreme; learning as exhibited behavior is at the other. The chasm between gives ample room for widely varying grains of analysis. I won't pretend to know why, in principle, anyone might choose a particular grain, but I suspect it has something to do with the community to which we make a commitment. If we commit ourselves to a community that values detailed functional explanations, then we should find value in Andy's orientation to computational theories. If we commit ourselves to a community that values imagery and metaphor, then Andy's orientation might feel too constraining. If

[4] For example, from one perspective teeth are structures; from another perspective teeth are processes of calcium formation. The two views differ by whether we take time into account. Even then, if we take time into account our unit of measure will affect how we think of teeth.

we commit ourselves to a community that values immediate, practical action, then Andy's orientation might seem irrelevant.

What a theory of learning is about is also dependent on our vision of what is to be learned. If we think that mathematics is applying rules for making marks on paper, then we will end up with Buggy-like theories of learning (Brown & Burton, 1978; Brown & VanLehn, 1981; Lewis, 1981). I have said enough about the small educational value of such theories (Thompson, 1989). I must say, however, that there is a cultural heritage in the United States of which we must become reflectively aware, and this is the heritage that elementary mathematics is ultimately about calculating. Our theories of mathematics learning will be hamstrung if we incorporate this heritage unthinkingly.

Finally, I take issue with one matter in Andy's paper. Theory-building in the physical sciences differs categorically from theory-building in education, and the difference has implications for how we respond to Andy's call for determining "what we know for sure." Physicists don't ever suspect that nature acts intentionally. Mathematics educators almost always assume learners act intentionally. We could say that intention is a natural characteristic of self-regulating systems, and thus kids differ from atoms only in their magnitude of complexity. We could, but it doesn't help. I see no way to theorize about learning without somehow framing the activity within personal experience. The trick is to reflect on where personal experience frames one's theories. Andy's suggestion to try finding why and when our propositions are true and false seems a promising mechanism for such reflection.

When is theory useful?

Andy alluded to how we often hear "theory" denigrated as if it has nothing to do with practical life. This may be most true of school teachers and undergraduate education majors, and it may be true of a larger number of our colleagues than we would like to admit. I have asked more than a few generalists who teach *Theories of Learning* courses to prospective teachers if they (the generalists) could teach algebra, or calculus, or differential equations given what they know about learning. "Algebra, perhaps, but not calculus and what is differential equations?" The teacher must rely on personal expertise in the subject. But what of the students sitting in this course, who do not have subject-matter expertise? Can we expect them to have a high sense of relevance of the course's content for their future lives as mathematics teachers?

In many respects I fail to see how theory can be useful to one who views "theory" as something out there, to be studied as an object in and of itself. If theory is to be productive for you, it must be *your* theory. This does not mean that you must construct it from scratch, or in absence of conversation. It means that the principles by which you observe and reflect are of necessity your principles. They cannot be propositions outside of your thinking. The distinction I have in mind is the same as the distinction between simile and metaphor. To think 'simile-ly' you have two things in mind, relating them analogically. To think metaphorically, you have one thing in mind, and you see it having characteristics which under other circumstances you wouldn't see. Theoretical thinking is metaphorical. Put differently, you have a theory when you assimilate the domain of interest to it. That's the way you see the world. *Useful* theory is "a light to the eye and a lamp to the feet … an organ of personal illumination and liberation .... [its value] consists in provision of *intellectual instrumentalities* to be used by an educator" (Dewey, 1929, p. 29).

Perhaps it is a matter of orientation as to what makes a theory useful. My orientation has been influenced by Les Steffe, who makes a strong distinction between mathematics *for* the learner and mathematics *of* the learner (Steffe, 1988). If one of our axioms is that we start where the learner is and build from there, then it follows that we must be able to think as if we were the learner. Thus, a theory of mathematics learning is useful to me when I can follow a paraphrase of Wyeth's exhortation: "Don't describe the child. Become the child!" This act of becoming, this attainment of coherent empathy, is only possible through theory. Without theory we are constrained to see children only as we *see* them; without theory we are constrained to hearing them only as we *hear* them. We can reflect on our mathematics to make it coherent, but without theory we cannot reflect on nor make sense of the coherence of children's mathematics. Reflective empathy is theoretical; theory building in mathematics education is the construction of reflective, analytic empathy.

Whence theory?

We sometimes hold the counterproductive view that theory comes from theoreticians. We all make theory. But of what do we make theory? Not from data, as Andy has already said. We have the freedom not only to build theory *of* practice, but to build theory *from* practice. Here I defer to John Dewey:

> The sources of educational science are any portions of ascertained knowledge that enter into the heart, head and hands of educators, and which, by entering in, render the performance of the educational function more enlightened, more humane, more truly educational than it was before. (Dewey, 1929, p. 76)

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