JMB JOURNAL OF MATHEMATICAL BEHAVIOR, **16** (2), 145–165 ISSN 0364-0213.

Clinical Interview Methods in Mathematics Education Research and Practice

ROBERT P. HUNTING
East Carolina University

Use of clinical interview methods in mathematics education research and as an assessment strategy in the mathematics classroom are contrasted. Differences and similarities between roles of researcher and practitioner are outlined. Uses of clinical interviews in research and practice are discussed by focusing on issues of how one prepares to administer an interview, kinds of tasks found to be most useful, kinds of questions one should ask, and how one should respond to students' answers and questions. A sample interview between an experienced teacher and a fifth grader is exhibited to illustrate the need for significant tasks, a sound pedagogical content knowledge base, in addition to interview skills and techniques.

The practice of formal mathematics assessment in classrooms has been driven primarily by a desire to monitor standards, provide accountability measures, and to improve reporting. As Cronbach (1963) has pointed out, these purposes are not the same as that of improving learning and teaching. Assessment data collected using conventional approaches is of limited use to teachers because important information about students thought processes needed to plan effective teaching strategies is masked, suppressed or ignored. The advantage that clinical assessment methods have over instruments designed to serve administrative regulation is that the data source (the student) and the data analyser and interpreter (the teacher–clinician) can engage directly in interactive communications. The teacher–clinician "reads the play" as the play proceeds.

Renewed interest in clinical approaches to assessment of learning in mathematics have coincided with recent emphases on *action–reflection* models of teaching (see for example, Schon, 1987) and orientations to psychological testing that admit more qualitative approaches such as dynamic assessment (Feuerstein, 1979; Gupta & Coxhead, 1988; Lidz, 1987, 1991) and individualized assessment (Fischer, 1985; Frederiksen, Glaser, Lesgold, & Shafto, 1990). A breakthrough in mathematics assessment occurred in the 80s when several authors rediscovered Piaget's clinical interview techniques (Donaldson, 1978; Ginsburg, Kossan, Schwartz, & Swanson, 1983; Hughes, 1986; Labinowicz, 1985). Labinowicz' textbook, in which he reported in detail the responses of young children to clinical interview tasks, was a significant advance.

Also during this time clinical research methods for investigating aspects of children's mathematics learning based on the tenets of constructivism have gained acceptance (Bell,

Direct all correspondence to: Robert P. Hunting, Mathematics Department, 129 Austin Building, East Carolina University, Greenville, NC 27858-4354.

1993; Pitkethly & Hunting, 1996; Saenz-Ludlow, 1994; Simon & Blume, 1994; Steffe & Cobb, 1988; Thompson & Thompson, 1994). These methods have their roots in Piaget's *méthode clinique* and the Vygotskiian teaching experiment. Not only have these methods made possible development of theory to explain the individual cognitions of children but also theory that accounts for the social context in which learning takes place. Central to these methods is recognition of the role of language and the importance of clarification of meaning as researchers ask questions and pose problems, and children talk about their mathematics and explain their actions. What makes these methods particularly significant for schooling is that teachers can apply adapted problems and tasks devised originally for research purposes, and with assistance, begin to make connections with theory and their own practice.

Clinical methods and tools are one of a range of assessment alternatives world-wide that are being trialled and evaluated in efforts to improve student learning of mathematics (Anastasi, 1990; A.C.E.R., 1994; Clarke, 1988; de Lange, 1987; Izard & Stephens, 1992; Leder, 1992; N.C.T.M., 1995; Niss, 1994a; 1994b; Romberg, 1992; Webb, 1993). Other alternatives include student portfolios and journals, investigations, open-ended questions, observations, performance tasks, and student self-assessment (Grouws & Meier, 1992).

CLINICAL INTERVIEWS AS RESEARCH TOOLS AND AS TEACHING TOOLS

Before discussing similarities and differences in the ways clinical interviews are used by researchers and by practitioners, I would like to address the prior question of what makes researchers and practitioners different. What characterises the roles of researcher in the university, and classroom teacher in the school? It could be argued that a major difference is similar to that which distinguishes a scientist from a technologist. The scientist is interested in working toward a better understanding of the world; the technologist is concerned with alleviation of an everyday problem (Cronbach, 1989). The scientist/technologist distinction applied to mathematics education research and practice is problematic. Mathematics teachers must address the problem of how to advance the mathematical knowledge of students placed under their care, both individually and collectively. The researcher works to build and test theory about mathematics learning and teaching in a more general sense, searching for explanatory patterns and principles, anomalies, and alternative ways of conceptualising problems in the field. But in education there are rare occasions, if any, when a solution to a problem can be handed to a teacher, like a new light bulb to replace one that has failed. Children's learning difficulties cannot be fixed in the same way that everyday tools might. As von Glasersfeld (1995) has argued, "Teachers must never be seen, nor indeed ever see themselves, as mechanics of 'knowledge transfer.' Rather, they should feel and act as the intuitive helpers who, in Socrates' words, play the role of the midwife in the birth of understanding" (p. 383). I am not the first to argue that good teachers are just as much involved in the formulation, revision, and application of theory, and that all teachers are prone to "see" what they believe, as are researchers. They must, however, act as facilitators, guiding students to discover for themselves the mathematical truths.

Similarities between the essence of research and practice cannot be under-estimated. However there is a difference in expectation between the researcher and the classroom teacher. The researcher is expected, by dint of specialized training, to make a contribution to a corpus of professional knowledge in ways that are accessible to the wider community, and in general the researcher has resources available to facilitate that mission. The practitioner wants to facilitate learning in children. The practitioner aims to make a difference—to effect change—at a local level; the researcher aims to make a difference at a more global level.

Another difference has to do with what might be called focus and the problem of context. The researcher is trained to isolate a question for investigation, and at the same time begin to understand it in relation to an existing knowledge base. The context for framing the problem is both collegial, and grounded in written contributions of others working on similar relatively focused problems. The practitioner is supposedly prepared, and certainly required, to deal with multiple streams of complex interrelated phenomena—physical, social, emotional, cognitive—and dynamic interactions across many levels in the teaching process including children's individual maturity, self esteem, and personality traits, their ways of responding in various group settings for different kinds of learning activities at different times of the day, according to unique management expectations and rituals which vary from teacher to teacher. A student's response to a mathematical task or question, and the teacher's interpretation of that response, is embedded in the thick soup of the classroom environment and community. In the classroom there is precious little time to reflect or focus on factors within direct control of the teacher that would promote learning. While the teacher is often having to make instant responses, the researcher has more time to carefully consider the data.

A difference affecting the readiness of a person beginning to learn the skills of interviewing, is mathematical sophistication. The researcher has an awareness of a particular topic, has defined an interest in the topic, and should be quite well mathematically prepared, whereas the practitioner may need further review of the linkages and relations between and within topics and concepts. Further, the researcher may be somewhat more conversant with pedagogical content knowledge vis-a-vis a specific topic or question of interest, due to access to and the study of literature—both curriculum and research, at least at a theoretical level.

In the discussion to follow I will attempt to draw out distinctions and similarities in the ways that clinical interviews are used by researchers and by practitioners. I will structure the discussion around a number of practical questions. These include issues of how one prepares to administer an interview, kinds of tasks found to be most useful, kinds of questions one should ask, and how one should respond to students' answers and questions.

THE INTERVIEW PROCESS

How Does One Prepare to Administer an Interview?

There are two major aspects of preparation common to both the cases of researcher and practitioner. First, there are interview techniques and skills, which need to be developed. Second, there are underlying assumptions upon which good interview practice is based. It is my contention that techniques and skills need to be embedded in an epistemology. Two important differences distinguish how a researcher and a practitioner might prepare to administer a clinical interview. First, for the researcher, choice or development of task and its associated protocol, is of paramount importance. Second, the researcher will typically have an interest in a fairly well defined question, so that the domain of focus will be narrower. Data will depend on the quality and incisiveness of task, so a fair amount of time is devoted to its trial and refinement. The practitioner is more interested in broader issues of understanding the mathematical abilities and competencies of students. Practitioners will expect to use tasks developed by others as tools to clarify understanding. There is a third and crucial difference. Researchers know that they don't know, or don't know well enough; teachers often feel that they should know, and moreover, if given the opportunity, should demonstrate what they know by telling or showing the student. The challenge for the teacher is to undergo a reconceptualization of role, necessary for effective clinical assessment, and for the quality of on-going assistance that may be provided. Teachers must become learners.

I believe it is helpful to make explicit for teachers certain assumptions and structural features that undergird the administration of clinical interviews. These include consideration of the role of language in the communication process, the observer/data collector as an integral part of the experiment, the interview as an opportunity to gather evidence and construct a model of the student's mathematics knowledge, the nondeterministic nature of the interview process, inherent dyadic power asymmetry and cue pattern searching. Upon such bases many interview techniques and skills rest.

The Role of Language in the Communication Process. A central assumption of the transmission view of teaching and learning is that meaning is inherent in the words and actions of the teacher, or in objects in the environment. A constructivist analysis of the communication process offers an alternative explanation to teachers of mathematics. An individual's conceptual structures must be inferred from their actions and verbal utterances. Communication can break down because the mental images and schemes I have learned to associate with particular terms, expressions, or physical materials may not coincide with the mental images and schemes evoked in the person with whom I am communicating. Issues of language, meaning, and communication have been addressed by von Glasersfeld (1995), and the problem of physical materials as representations of mathematical concepts have been discussed by Cobb, Yackel, and Wood (1992) and Hunting and Lamon (1995).

Observer as a Subjective Participant in the Experiment. It is a concern to some teachers that in the clinical interview approach the interviewer apparently abandons all semblance of objectivity. A student's response cannot be a reflection of their true ability if the interviewer is likely to rephrase the question, substitute different terms, offer a simpler problem, or worse, ask a leading question. In traditional test situations one refrains from influencing or engaging with students because to do so would threaten test score validity. One wants to be able to say that a student's score on a test is a true indication of their knowledge of the content being tested. Any other influence likely to contribute variance to the test score should be minimized. Researchers who use clinical methods routinely video tape their interviews. The advantage here is that it is often (but not always) possible to observe, reflect upon, discuss, and test alternative interpretations of a student's response, *by taking into consideration relevant counter-responses made at the time by the interviewer.* A feature of our training program is to require teachers to video themselves conducting an interview, critique their technique based on the literature, then repeat the interview with another child—again videotaped—and report on how they have improved in technique, and what aspects proved to be resistant to change. Just like good teaching, the clinical interview is a dialogue, and the interviewer is uniquely integral to the assessment process. Good teaching entails a strong evaluative aspect. Just as the researcher formally records interviews for later analysis and reflection, so the teacher–clinician will later replay problematic segments of an interview in the mind's recorder. It is through such a process that reflection begins to happen, along with the possibility of deeper understanding of a student's mathematical reality.

Nondeterministic Nature of the Interview Process. A related concern is associated with the interactive nature of the dialogue. There are certain features of a clinical interview task that remain invariant across students. First is what we would call the physical materials often (but not always) associated with a task. Second, the initial statement or presentation of the problem, usually concluding in a question for the student or an invitation to respond. However, after the first response of the student, an interview is likely to take any of an infinite number of paths. Many interview tasks contain suggested follow-up questions, in anticipation of commonly observed responses. A particular feature found in interview protocol is a request for the student to explain an action, or solution. Such a strategy may elicit from the student important clues about the quality of thought involved. Although possible, it does not make sense therefore to compare students' responses to a task quantitatively as right or wrong. Such a concern is born out of a measurement tradition focused on product outcome along with high reliance on score meaning. When the assessment method is oriented to uncovering the processes by which students arrive at problem solutions, efforts are made to give students every opportunity to explain their thinking. We are beginning to document and catalogue commonly occurring solution strategies to interview tasks. Our aim in conducting such process analyses, is to provide teachers with interpretive comment, as well as a broader spectrum of data against which they can relate their experience with particular students (Pearn, Doig, & Hunting, 1996).

Interviewer as Modeller. When a teacher poses a problem or presents a task to a student, he or she has in mind what the task is about, and how the associated materials might facilitate a solution. The teacher will also, perhaps unfortunately, have in mind a solution strategy indicative of understanding on the student's part. The danger here is that the student may well respond to a different problem compared to the one the teacher thought was presented. The critical stance of the interviewer is to set aside preconceived notions of the significance of a student's response until there is sufficient evidence available that the problem as interpreted by the student is indeed the problem as intended. As von Glasersfeld (1995) warned, "If the meaning of the teacher's words and phrases has to be interpreted by the students in terms of their individual experiences, it is clear that the students' interpretations are unlikely to coincide with the meaning the teacher intends to convey" (p. 182). Out of the linguistic and nonverbal communications that transpire in the course of an interview, the interviewer attempts to construct a model of the student's mathematical knowledge. Steffe (1995) identified such models as *second order* models to distinguish them from hypothetical models "the observed subject constructs to order, comprehend, and control his or her experience (p. 495). Such are first order models. There is no way of seeing directly into the student's head and observing the mathematical machinery pulsing and twitching there. On the other hand it is not true that the interviewer has absolutely no clue as to some of the possibilities. The experienced teacher will have developed an "abstracted mathematics curriculum" (Steffe, 1990), which would include a set of general expectations for a student of a particular age and year level. An *abstracted mathematics curriculum* is not a written document involving elaborations on the scope and sequence of expected learning targets. Rather, it is a fairly general mental model, informed by experience of teaching individuals, groups, planning instruction, consulting textbooks, accommodating ideas from new research literature, discussions with professional colleagues, and so on. It is the long-term goal of the mathematics teacher to improve the sophistication of such a general model. Individual encounters with students will contribute to the refinement of a general model. Degree of refinement of a general model will enable the teacher-clinician to make more accurate inferences, hypotheses, and ask more incisive follow-up questions in the quest for particular models of the various components of an individual's mathematical knowledge.

Power Imbalance. The student is, initially, a conscript; perhaps unwilling, certainly apprehensive. The interviewer seems to hold all the cards. An environment of mutual trust and respect is necessary in order for the student to relax and talk freely about the task. In fact, Ackerman (1995) argued that "for the clinical setting to work at all some kind of agreement—or contract—needs to develop along the way" (p. 346). Students have learned that school is a place where extrinsic rewards and punishments motivate progress. If you want the teacher off your back, then do what they want. Students lacking confidence in their personal strategies for reasoning through a mathematical problem, or lacking the necessary conceptual foundations (they "just don't get it"), will seek any sign from the interviewer that they are on the right track as they attempt to solve a problem. On the other hand, interviewers are keen to give as much encouragement as possible, because they want students they interview to demonstrate what they do know rather than what they don't know. They also want their students to experience success, and reward success. Interviewers can inadvertently lead students to make correct responses, and students are good at sleuthing information desired. The teacher as power-guru can be manifest in the urgency with which follow-up questions are posed, denying the student time to think about a task.

Classroom teachers have the potential to make excellent interviewers. A professional development program incorporating practical and theoretical preparation (Hunting & Doig, 1997) might include the following features:

- Review of goals of teaching mathematics. For elementary teachers in particular, we have found it useful to contrast similarities and differences between language learning and mathematics learning;
- Consideration of constructivist versus behaviorist theories of learning, with particular emphasis on the communication process, role of meaning and interpretation in comprehension of verbal utterances and written text, the need to clarify intention, and the need to see the problem from the point of view of the other;
- Consideration of reasons students turn off mathematics; why they fail;
- Defining features of a clinical interview, observation of a clinical interview, and discussion of interview techniques;
- Features of clinical assessment instruments;

- Opportunities to consider research rationales underpinning clinical assessment tasks (Hunting, Gibson, Pearn, & Doig, 1995);
- Case studies of school-based implementation/intervention programs;
- Intensive supervision in development and practice of clinical interview techniques with children;
- Opportunities to interpret students' mathematical behavior—both individually and with the assistance of an observer, other participants and supervising staff;
- Opportunities to consider diversity of responses to specific tasks, and discuss their implications, and possible assistance strategies;
- Development of skills in reporting interview outcomes for use by a teacher who will use the report to plan an intervention or enrichment program, development of skills in interpreting others' reports;
- Development of action plans targeted at assisting students with specific mathematical learning difficulties;
- Opportunities to apply clinical assessment methods in a local school setting; and
- Consideration of alternative assessment tools and strategies.

What Kinds of Tasks are most Useful?

Any mathematical problem, task, exercise, or test item may be used in order to engage a student in action and dialogue with the outcome of giving the teacher a window into that student's thinking. The important thing is to introduce some stimulus out of which a conversation may be initiated. There are, however, criteria implicit in choosing or developing tasks to be used in clinical assessments of mathematics knowledge. Considerations in selecting include:

1. Time Availability. Time needed to be set aside for an interview depends on the age of the student. With five to eight year old children that time might range from 10 to 20 minutes; with 10- to 12-year-old children that time might range from 35 to 50 minutes. Experienced interviewers are able to recognise when children have reach the limit of their concentration. Squirming and fidgeting is one indicator. From the point of view of the practitioner an overriding aim of a clinical interview is to maximise the information in the time available. Our experience has been that of attempting to include more tasks than can reasonably be administered in a given time. We have developed "short" versions of task sets. Supplementary tasks from a larger pool of tasks may be administered in subsequent interviews as appropriate.

2. Prior Information. If the interview is to be used for screening purposes, as might be the case in assessing all children at a given year level to identify those to be included in an enrichment program, then a "short" set of tasks might be used. A more extensive set of tasks may be administered to children identified as a result. Alternatively, other forms of assessment, including teacher observation and inspection of students' work, may lead a classroom teacher to identify a group of students to undertake an enrichment program. In that case initial sessions of the program may be devoted to intensive clinical work using a wide range of tasks as a means of determining the nature of instruction to be implemented.

3. ***Novelty.*** It is desirable that a task engage the student's interest. Students will be more likely to become interested if the task is novel in its presentation or context.

4. ***Context.*** The task should require the student to bring to bear mathematical thinking but should be framed in a setting that is realistic to the student, if possible.

5. ***Materials.*** Some tasks require the student to consider physical material; others do not. Tasks that involve manipulatives provide greater opportunity to observe students' actions along with their verbal explanations and comments, but entail greater risk of alternative interpretation to that intended. We routinely provide pencil and paper for students to use to record their solution attempts. A particular difficulty with tasks that attempt to assess students' spatial knowledge is dependence upon two dimensional diagrams and figures to represent three dimensional objects and situations. One might be tempted to infer knowledge of three dimensional spatial objects and relations but in fact what is assessed is a student's ability to interpret two dimensional representations of three dimensional objects and relations.

6. ***Flexibility.*** Tasks can cater for a broader band of student ability by incorporating easier or simpler subtasks. For example, a task we use for upper elementary students asks them to determine the fraction two thirds of 12 swap cards (a set of 12 cards is made available). For a student who has difficulty with two thirds the interviewer has the option of restating the task using the fraction one third or if necessary, one half. If the student successfully responds to the easier task, the original task may be posed again.

7. ***Research Base.*** Much curriculum content and sequence has its justification in logical analyses of elementary mathematics by mathematics educators and mathematicians. What has begun to change over the past 25 years is deliberate effort on the part of the research community to re-evaluate traditional curriculum in the light of results of studies investigating the mathematics of children. It is an advantage to examine research into the learning of mathematics because the tasks used in that research have usually been subjected to a degree of rigorous analysis, there is normally a discussion of supporting literature addressing what is known about the topic, and certainly some conclusions and recommendations based on the findings. In addition it is possible that new or alternative conceptualizations about how a topic might be approached have been considered. Assessment tasks that are grounded in research have potential to offer new perspectives on student's mathematical development, and may lead to curriculum innovation.

8. ***Curriculum Linkage.*** To the extent that curriculum statements (as e.g., Australian Education Council, 1990; Department of Education and Science, 1988; NCTM, 1989) reflect a social consensus about mathematics goals, targets and learning outcomes, it behooves developers of clinical interview tasks to locate their tasks within those broad frameworks. Checking whether a task bears some relation to a mandated curriculum standard does not necessarily legitimize the task, or the curriculum. However such an exercise can prevent oversight of an important concept or skill.

What Kinds of Questions Should One Ask?

Questions are a key feature of clinical assessment tasks because the nature and timing of a question is critical for the interviewer. In general questions should

- be open ended so that students are allowed some freedom to choose their own preferred ways of responding
- maximize opportunity for discussion or dialogue so that thought processes can be revealed, and
- allow both student and interviewer to reflect on their respective thought processes.

An interview task typically commences with a statement from the interviewer in which a problem is presented. Problem presentation concludes with a question inviting the student to respond. Presentation of the initial task, including the first question, is fairly straightforward, since the form of presentation has usually been scripted carefully to make the problem statement clear, and as far as possible obviate unnecessary or unfamiliar terminology. Tasks designed for research studies will commonly continue to prescribe subsequent questions, sometimes incorporating elaborate pathways and branches. Such protocols are the result of careful trialing and refinement of questions to address a specific and focused research problem. In the case of the practitioner, task sets represent a broader domain of mathematical knowledge. Where tasks have been adapted from research, advantage can be taken of detailed follow-up question sequences. There is less pressure on the practitioner to "stick to the script," so that follow-up questions are suggestions rather than requirements. However, many tasks have no such background. In these cases, one generally cannot know exactly what the second or subsequent questions will be, and the best general guide is to use common sense. However there are some general forms of follow-up question used by experienced interviewers:

Can you tell me what you are thinking? This question is useful after about 10 seconds of silence where it is not certain that productive mental activity is taking place.

Can you say out loud what you are doing? When a student seems to be engaged in thought, after giving a short time for this—perhaps 10-15 seconds—the interviewer may interrupt. Indicators of activity include inaudible utterances, scratch work on paper, motor activity such as tapping, eye and other body movements.

Can you tell me how you worked that out? How did you know? How did you decide? A student may respond with an answer to a problem without any apparent clue as to the way the answer was obtained. These questions are intended to convey to the student that you are interested in how the result was determined. As such it is designed to encourage a verbal explanation.

Was that just a lucky guess? If the student makes a response but does not give an explanation, then this question often has the effect of putting the student at ease and relieving tension. Sometimes in an effort to obtain information students will respond with the first thing that comes into their head. Students are generally happy to admit guessing.

The other day another student told me... If there are grounds for supposing that the student isn't confident about the solution offered, or the interviewer wants to test the strength of a conviction, an alternative solution from a neutral and anonymous third party may be proposed for consideration. The advantage of attributing the alternative solution to a third party is that the student could feel it in his or her best interests to agree with a view emanating from the interviewer, just because the source of that view has power and status in the situation.

Do you know what ___ means? Success on a task may depend on knowledge of a particular term used in presentation of the problem. Potentially problematic vocabulary can be nullified by clarifying the meaning of the term. Teachers being teachers have an uncontrollable urge to teach. Should a teacher explain a point during an interview? The answer to this question rests on whether the teacher primarily intends to assess the status of the student's mathematical knowledge. It is not wrong to provide a student information. In fact, there are benefits in seeing how far the student is able to progress on the basis of some assistance. It may be that the information provided allows the student to incorporate other knowledge previously untapped. It is worth bearing in mind that the interview itself is a learning experience for the student. The extent to which the teacher digresses into a didactic frame during an interview will dictate how much progress will be made through the interview given the time available. We generally discourage teachers from digressing during formal training.

Do you know a way to check whether you are right? Problem solutions, particularly those involving basic arithmetic operations, can be checked by means of estimation, rounding, or the appropriate inverse operation. Encouraging checking provides another window into a student's depth of understanding.

Why? In response to an explanation a student may make an assertion. Asking why is a sensible way of encouraging further explanation.

Pretend you are the teacher. Could you explain what you think to a younger child? How would you explain? Encouraging children to formulate viewpoints or design settings for younger children provides an opportunity to capture their understanding of a situation or problem.

How Should One Respond to Students' Answers and Questions?

The overall aim of a clinical interview is to create a relaxed atmosphere and establish a relationship of trust. This is a greater challenge for researchers, because they typically present to students as strangers. The practitioner will be known to the student, and will have established at least a reputation if not rapport. By reputation, I include aspects such as personality, attitudes and response patterns to a range of student behaviors, including management style.

Early in the initial interview, prior to introduction of the first task, interviewers might explain to students that they are *not* interested in whether an answer is right or wrong. What they are interested in is how the answer is obtained. Various story lines can be advanced, such as the interviewer needing to learn more about how children think about mathematics, how students sometimes know how to do a problem, but have trouble writing it down, and so on.

But the manner in which an interviewer responds to a student's answers and questions is critically important. It can be assumed that in general the student has a desire to please. In most classrooms social norms work to reward appropriate behavior and disapprove inappropriate behavior. Inexperienced interviewers have a great deal of trouble coming to terms with a theoretically desirable set of social norms different to those that usually reign supreme in classrooms. In clinical contexts the interviewer attempts to respond in neutral ways regardless of the correctness of a response. This worries teachers a good deal because they feel they are morally obliged to tell students if they begin to stray from the conventional wisdom. The problem in interview contexts is that if students do not feel confident about a problem, they will tend to look first for external clues rather than rely on their own resources, thus closing out opportunities for processes of thought to be elicited. Responses of encouragement such as "good," "great," "uh-huh," and "okay" are neutral if used consistently and without variation in emphasis (e.g., when the student achieves a solution after some effort).

Another norm of regular classroom life is that one is not usually questioned in detail unless a misdemeanor has been committed, or an error has been made. Because requests for detailed explanations are traditionally associated with negative consequences, students are likely to interpret follow-up questions as indications that their solutions are unacceptable and in need of revision. Acceptable behavior in classrooms rarely attracts detailed analysis as to its origins and sources. Interviewers may tell students at the beginning of an interview that they are interested in how solutions to problems are determined, and being asked questions about solutions doesn't mean they are wrong. We are just as interested in how correct answers are obtained.

It is one thing to achieve even handedness with verbal cues. It is another to do so with nonverbal cues. Body language such as leaning forward or back, variation in intensity of gaze, eye movement, facial expressions such as furrowing of brow and raising of eyebrow, position of arms and hands, may all be inspected by the student for patterns indicating success or failure; approval or disapproval.

SAMPLE INTERVIEW

The following transcript is part of an interview with Simpson, who was 11 years, 0 months, and in Year 5, at the time. In the transcript that follows he appears as S. Tasks given in Parts 1 and 2 of this interview are from Section 4 of the Initial Clinical Assessment Procedure in Mathematics (ICAPM), Level B (Hunting, Doig, & Gibson, 1993). Section 4 deals with the topic Fractions, Ratios, and Decimals. A rationale for these tasks is provided in Hunting, Gibson, Pearn, & Doig (1995). The interview segment has been split into three parts. Parts 1 and 2 deal with Simpson's responses to each of two tasks. Part 3 details further conversation resulting from a spontaneously generated task by the interviewer. The interviewer was an experienced teacher. In the transcript that follows we will call her T.

Nonverbal behavior and indication of length of time between action is indicated in parentheses. Comments on relevant aspects on this interview segment appear in the body of the transcript in square brackets.

Simpson had just successfully responded to several questions about the place value of digits in four-digit numerals.

The next task offered to S was Level B Task 4.2 (see Figure 1).

> "Here are 12 swap cards that you and your friends have collected.
> You collected 2/3 of these cards. How many cards did you collect?"
>
> If successful, ask:
> "How did you find that out?"
>
> If necessary, ask S to cover the cards with a hand.
>
> If unsuccessful, ask:
> "What is 1/3 of these cards?"
>
> If unsuccessful, ask;
> "What is 1/2 of these cards?"

FIGURE 1. Protocol of Task 4.2

Part 1: Task B4.2

T: Here are 12 swap cards that you and your friends collect, okay?
[T checks that S understands the initial part of the problem.]

S: Okay.

T: Now, you collected 2/3 of that number of cards (places the cards out in two rows of six).
How many did you collect?

S: Pardon, what was the—two-thirds?
[S asks for clarification. Perhaps he did not hear or does not know how to respond.]

T: Yes, how many of those did you collect? Oh, that hasn't even got a picture on it. Okay. Two-thirds you collected.
[T repeats the task.]

S: (Point counts silently) is that 11?

T: You had better check then.
[T encourages S to verify the number of cards for himself.]

S: Okay, one, two, three,—(counts each card again)—11, 12.

T: Okay, that's 12.

S: Um!

T: You collected two-thirds.

S: That's half I think. That's half of how many's here.

T: That number we want (places flashcard with 2/3 in front of S).
(Silence for 3 seconds) What does this mean to you? When I show you that number (indicates flashcard) what does that mean to you?
[T decides to probe Simpson's response further. Her decision to do this entails some risk, because by pressing him in this way may possibly cast doubt in Simpson's mind concerning the acceptability of his original response. She is rewarded for persisting, because Simpson reveals the logic of his thinking and the meaning he has for this fraction—the numerator means the number of groups, and the denominator means the number in each group.]

S: There's—I have two groups and there's three in the group.
T: Okay, so the top number means the number of groups—
S: Yep.
T: —and that means how many there are in each group (pointing to the digit three on the flashcard)?
S: Yep.
T: Okay. Alright!
S: (Points and counts six of the cards again) that's all mine.
T: Okay, so that group there is two thirds?
S: Yep.
[T is not sure precisely what to do next. She hears what S has said concerning two thirds, but needs time to consider its ramifications. Temporarily thrown by the unexpected response and its reasonableness, she omits the follow-up task options and moves onto the next task—B4.3.]

Part 2: Task B4.3

Task B4.3 focuses on constructing a whole from a part 3/5 the protocol appears in Figure 2.

T: Okay, this time we're going to only have those six (gathers up cards leaving six on the table), and that represents... you and your friends collect those cards and you get six, okay?
S: Yeah.
T: And that's three-fifths of all the cards. Three-fifths. How many cards were there altogether?
S: What do you mean by three-fifths
[S is relaxed enough to ask for clarification.]
T: (Writes 3/5 on a sheet of paper.) three-fifths.
[T doesn't 'tell' S what he wants to know. But she does assist by writing the numeral for the fraction on a sheet of paper.]
S: There has to be 10. Because there's—not five in—there's only one five. There's not three of these so there will have to be 15 in each group, like all of this there should be 15—
[S attempts to use his interpretation of fraction in this new task situation.]
T: To start with?
S: Yeah.
T: Or 15 cards is the total amount?
S: Yeah, 15 cards is the total amount.
T: How did you work that out?
S: Cause five represents how many is in a group.
T: Right.
S: And three represents how many groups of—
T: So there are three groups of five which make 15?
S: Mm.

> "Suppose you and your friends collect swap cards and you get these cards. And these cards are 3/5 of all the cards. How many cards were there?"
>
> If S is successful, ask:
> "How did you find that out?"
>
> If S is unsuccessful, repeat the task above using 3 cards.
>
> If still unsuccessful, take 5 cards and discuss 3/5 of the 5 cards.
>
> Repeat the task above with the 5 cards.

FIGURE 2. Protocol of Task 4.3

T: Okay (collects up cards). If I said to you there are three cards (places three cards down) and that's three-fifths, how many would be in the whole— how many cards did you start with?
[T asks the fall-back questions suggested in the Level B4.3 task protocol.]
S: Three, and that would be a fifth (indicating one of the cards).
T: Okay (puts out two more cards). Say I was to give you five cards, and ask you for one-fifth, what would that mean to you?
S: You can't do it. I'd have to give you—that (pushing out the middle card) and a quarter of that card (indicating adjacent card) or something.
[It is not clear what S is thinking here.]
T: That and a quarter.
S: No no, not—two-thirds.

Part 3: Sharing between five

T: Okay, let's start again. (Gathers up five cards into a single stack and offers the stack to S.) I'm going to give you five cards this time; and I'm going to ask you to share it between five people.
S: (Separates out stack of cards.)
T: Good. If I was to give you 10 cards (gathers up cards and adds five more, placing them in a single stack in front of S) and ask you to share them with five people, could you do that for me?
S: Yeah (separates out two cards at a time and makes five piles).
T: This is my share (picks up pile of two cards), okay? If you were to describe to me what my share is, of that whole amount of cards, how much would I get?
S: A fifth. And you've got two cards.
[T decides to check further the strength of Simpson's apparent insight.]
T: And if that's your share (points to pile in front of S) how much have you got?
S: A fifth.
T: Okay, and if you've got all of that (picks up two piles of two cards leaving three piles on the table) how much have you got?
S: What, all of these (waving hand over the three piles)?

T: Uh-huh.
S: Three-fifths.
T: Okay.

Simpson had a reasonable but inappropriate understanding of fractions such as two-thirds and three-fifths. (I hesitate to say fractions period, since Simpson seemed to have a deeper understanding of one-half). Somehow he had confused a conventional interpretation of common fraction notation—that the denominator indicates how many groups into which the whole is partitioned, and the numerator represents the number of such groups to be considered. While the teacher elicited data from Simpson which unambiguously indicated how he understood two-thirds, she did not, right at that point, know exactly how to interpret Simpson's response. So she used a well-tried technique in such circumstances—carry on with the next part of the interview with a view to attempting further clarification later. If insufficient data is available to assist the interviewer make a sensible or reliable interpretation, an alternative task or approach may be attempted at a later time. Later may mean a few moments, as illustrated in this case in Part 3 of the interview segment, or it may mean a few days.

Children's fundamental ideas about fractions appear to depend on the development of mental schemes for relating units. Students come to school with (relatively) sophisticated knowledge about units and how to deal with them. During primary schooling their ability to relate and coordinate various types of units increases. For example, young children have powerful systematic methods for dividing collections of items into equal shares. As their knowledge of whole number relationships develops, outcomes of sharing problems can be anticipated using numerical means. Children will also learn that similar quantities can be partitioned differently, with an inverse relation between size and number of parts. Simpson's problem was not so much a lack of "unit" knowledge—that is, fundamental operations for coordinating various quantities and their relationships. His responses to the tasks the teacher posed in Part 3 showed that he had not connected his knowledge of units with the symbolism and notation of conventional fraction instruction.

What was the nature of Simpson's "unit" knowledge that allowed him an opportunity to reconstruct his knowledge about two-thirds? Briefly, from an observer's perspective, in the Task B4.2 setting there were distinctly different types of units, and there were different ways of coordinating these units. The four unit types (Hunting, 1983) were: (1) the set of 12 cards, (2) each card as a single item, (3) the units of one-third, each consisting of just four cards, and (4) the unit of two-thirds, resulting from the composition of two of the one-third units. In Part 3 of this interview segment Simpson was able to relate three types of units—the set of 10 cards, each individual card, and the five equal shares. One way of determining the number of cards each of five people would receive would be to "deal" them out, using a systematic action sequence. We have researched this proto-rational number scheme in some depth (Hunting & Davis, 1991). Another way is to use whole number knowledge to predict the sharing outcome, and this is what we believe Simpson did. Simpson's behavior is consistent with a partitioning scheme where whole number relationships can be used to anticipate the outcomes of sharing problems. The key numerical relationship in this instance was $10 \div 5 = 2$ or $5 \times 2 = 10$. That is, Simpson had a sophisticated scheme for coordinating relationships between types of units. Developmental dependencies between whole number knowledge on one hand, and rational number knowledge on the other, are not well understood, but have begun to be addressed (Hunting, Davis, & Pearn, 1996).

The teacher knew more than the techniques of interviewing. While she had in fact departed from the "script," she contributed her own questions. These were relevant and fruitful. This teacher had an understanding of fractions based on equal shares. Depth of pedagogical content knowledge is a necessary foundation to which interview skill and technique can be fused. Apart from securing cooperation from the student, three necessary ingredients for successful interviews are significant tasks, a sound pedagogical knowledge base, and good interview technique. The teacher in this interview set up an obvious direction to follow in subsequent teaching sessions. Simpson was comfortable with the notion of equal shares of quantities, and he spontaneously used fraction language to describe the teacher's share after dividing 10 cards between five people. So in future instruction the teacher might sensibly devise sharing problems where fractional language and notation could be introduced and discussed. Simpson's inappropriate interpretation might be expected to persist for some time *along side the more robust interpretation based on sharing promoted by the teacher.*

How much did the teacher know about theories of fraction learning underpinning these tasks at the time of interview? She was certainly aware of the potential of sharing as a basis for fraction knowledge, and exercised remarkable skill in departing from the "script" of the ICAPM task sequence to assess Simpson's understanding of it. The critical thing is that knowing what is the research rationale for a clinical task doesn't guarantee that the context will allow that knowledge to become accessible in the moment. As we saw, the teacher was temporarily thrown by Simpson's unexpected but sensible response. The issue of what are efficacious ways to assist teachers make wise decisions in interview settings deserves more research. One strategy we have embarked on is to closely observe responses children make to tasks and questions, catalogue common responses, and provide interpretive comment based on research and the literature, where that is available.

We believe this teacher's knowledge about the significance of the task contributed to her eventual success finding a point of contact in sharing for instruction with Simpson at a later date. The process of communication, and the results of Sandra's interpretations of Simpson's responses provided powerful pedagogical clues for continuing a productive learning experience for this child.

CONCLUDING COMMENTS

Clinical interview methods acknowledge the important role of interpretation in the teacher's assessment of a child's learning. When a teacher engages in interactive communications with a child, that teacher is attempting to understand, from the child's point of view, what that child understands about a concept, problem or topic with a view to helping the child advance in understanding. Since the teacher cannot get inside the child's head to know first-hand what the child knows, he or she must necessarily make inferences about the child's knowledge. Traditional large scale testing creates a chasm between the student as data source and the data analyzers and interpreters. This void precludes opportunities to clarify a child's conceptual understanding through interactive communications, or make provision for informed interpretations of responses.

Over the past five years we have been engaged in development of clinical interview tasks, and providing advanced training in clinical interview methods for teachers. Development of tasks has involved application of novel validation criteria that we have devised specifically for this kind of assessment: content relevance and representativeness, theoretical validity, process analysis, and useability (Hunting & Doig, 1992). Content relevance and representativeness involves analysing national and international curriculum statements to determine links between each interview task and corresponding content cells. In addition, experts in mathematics education are asked to comment on tasks, task protocols, and sequencing. Theoretical validity entails providing rationales for tasks that have a basis in the mathematics education research literature. Process analysis involves building up profiles of response categories that indicate important information about the conceptual understanding of the student being assessed. Useability involves evaluations of utility, clarity, and strengths and weaknesses of the task sets by practitioners trained in their use. We are currently conducting process analyses of Grade 5 and 6 students' responses to tasks. Video data is being tagged, classified by task, and categorized. Transcripts of commonly occurring response categories are being prepared, and interpretations of responses developed using relevant research literature where available. The aim of the interpretive comment will be to assist teachers fit responses of individual children against a broader pedagogical framework.

A clinical interview approach to assessment is more than a set of techniques. To be successful, practitioners need to be both humble and wize; humble in the sense of being prepared to gain new insights into the mathematical learning processes of children, and wize in understanding the strengths and limitations of the method, and securing a deep knowledge of the pedagogy and research underpinning the tasks used. There is no recipe for guaranteed results. Interview skill does not come instantly. Even the most experienced clinician will occasionally be rendered impotent by a student's response, with no sensible hypothesis to test in the moment. There are good grounds for supposing that improved skills of listening and questioning gained from professional development in clinical interview methods will transfer to the conventional mathematics classroom. However this conjecture awaits empirical support. Should teachers lacking pedagogical content knowledge be encouraged to use clinical interview approaches to mathematics assessment? I say yes. Possession of pedagogical content knowledge, or an abstracted mathematics curriculum, is not an all or nothing affair. None of us know it all. Clinical approaches to assessment can open the door for teachers to begin to expand their experience of how children's minds work mathematically. Students should have every opportunity to participate in improved communication with their teachers. Clinical interviews allow students to teach teachers.

Is it too much to expect teachers to use clinical assessment approaches in classrooms? I don't believe so, provided two related obstacles can be overcome. The first obstacle is the cost associated with preparing teachers to be clinicians. The second obstacle is finding time to conduct assessments one-to-one with students. The first obstacle is surmountable. Empowering teachers with skills and tools is important. We need viable alternatives to traditional paper and pencil assessment methods. If it is true that you only get what you pay for, then it is worth investing more to teach better. Becoming a teacher–clinician is not going to happen overnight. Professional development can help to substantially reorient teachers by alerting them to the rationale and philosophy of clinical assessment, and allow them to gain practical interview skills and techniques, but that is only a start. Each clinician will begin to expand his or her experience base slowly, through on-going discussions with colleagues, and with access to new results from research. There needs to be recognition on the part of schools that advanced skills in assessing and assisting students' mathematics progress is important and worth encouraging. Teachers who have the skills might be used to assist other teachers identify and assist students needing special programs. Inevitably there will be resource implications. The second obstacle may appear more intractable. Schools traditionally organise students into class groups of 20-35 for instruction. It is difficult for a teacher to be able to wedge out time for significant on-going interaction with individual students. In elementary schools teachers have more flexibility because they generally teach the same class for the core curriculum at least. In Australia there is heated debate about the social consequences of withdrawing students from the classroom to participate in intervention programs. If a student is withdrawn to participate in an intervention program will that student be "labelled"? What will be missed from on-going regular instruction, and how will it be compensated?

There are more similarities than differences between the roles of researcher and practitioner, because both are deeply interested in how children think and learn. Both are committed to gathering and interpreting evidence about children's learning. The classroom practitioner needs to do these in order to make curriculum decisions that will facilitate the on-going progress of students. The researcher needs to do these as part of the knowledge generation process involving theory construction, testing, and reconstruction.

Clinical interviews are a means by which teachers can observe and interpret, that is, assess, the mathematical behavior of students. Clinical interviews also provide the bases for interventions in which explicit strategies, activities, and settings are designed to fit the current state of students' mathematics knowledge. Such interventions, springing from a deep understanding of the student's thinking, can help to propel them to deeper and more powerful conceptions. Teachers who excel at this, we propose, will not only use clinical techniques to appraise what is the nature of the knowledge of the student with whom they work, but they will also be able to provide opportunities for students to take advice, information, or hints, respond to questions, and to stand back from their own mathematical activity, reflect, and communicate their understandings.

The Assessment Standards for School Mathematics (NCTM, 1995) asserts that "teachers are the persons who are in the best position to judge the development of students' progress and, hence, *must be considered the primary assessors of students*" (emphasis mine, p. 1). To effectively judge a student's progress, in our view, means to utilize those assessment methods and techniques which will enable sound interpretations to be made. We claim that clinical interview methods using tasks that have a basis in the literature and research of mathematics pedagogy is one such method. Clinical methods suggest teaching approaches that emphasise questioning and listening, as distinct from telling. In addition, by virtue of the flexible nature of this approach, it is possible to account for students' thinking processes in addition to the results of that thinking. Even more fundamental is explicit recognition that mathematics knowledge is more than just the student's behavior in response to a problem or task. Assessment is about gathering evidence from which inferences about cognitive capacities and constraints can be made. Understanding the mathematical workings of children's minds is now a priority for teachers, schools, and systems, as well as for academic researchers. Partnerships between these stakeholders have much potential for reforming school mathematics.

Acknowledgment: This is a revised version of a paper presented at the Symposium on Interview Techniques for Classroom Researchers and Practitioners, Research Presession of the 74th Annual Meeting of the National Council of Teachers of Mathematics, San Diego, April 23-24, 1996.

REFERENCES

Ackerman, Edith. (1995). Construction and transference of meaning through form. In Leslie P. Steffe & Jerry Gale (Eds.), *Constructivism in education* (pp. 341-354). Hillsdale, NJ: Erlbaum.

Anastasi, Anne. (1990). *Handbook of psychological tests*, 3rd ed. New York: McGraw-Hill.

Australian Council for Educational Research (1994). *ACER educational resources catalogue*. Hawthorn: Author.

Australian Education Council (1990). *A national statement on mathematics for Australian schools* (draft). Canberra: Author.

Bell, Alan. (1993). Some experiments in diagnostic teaching. *Educational Studies in Mathematics, 24*, 115-137.

Cobb, Paul, Yackel, Erna, & Wood, Terry. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*, 2-33.

Clarke, David. (1988). *Assessment alternatives in mathematics*. Mathematics Curriculum and Teaching Program Professional Development Package. Canberra: Curriculum Development Centre.

Cronbach, Lee J. (1963). Course improvement through evaluation. *Teachers College Record, 64*(8), 672-683.

Cronbach, Lee J. (1989). Construct validation after 30 years. In Robert L. Linn (Ed.), *Intelligence: Measurement, theory, and public policy* (pp. 147-171). Urbana, IL: University of Illinois Press.

de Lange, Jan. (1987). *Mathematics, insight, and meaning*. Utrecht, Netherlands: Onderzoek Wiskundeonderwijs en Onderwijs Computercantrum.

Department of Education and Science (1988). *Mathematics for ages to 16: Proposals of the Secretary of State for Education and Science and the Secretary of State for Wales*. London: Author.

Donaldson, Margaret (1978). *Children's minds*. London: Fontana.

Feuerstein, Re'uven (1979). *The dynamic assessment of retarded performers: The learning potential assessment device, theory, instruments, and techniques*. Baltimore: University Park Press.

Fischer, Constance T. (1985). *Individualising psychological assessment*. Monterey CA: Brooks/Cole.

Frederiksen, Norman, Glaser, Robert, Lesgold, Alan, & Shafto, Michael G. (1990). *Diagnostic monitoring of skill and knowledge acquisition*. Hillsdale, NJ: Erlbaum.

Ginsburg, Herbert P., Kossan, Nancy E., Schwartz, Robert, & Swanson, David. (1983). Protocol methods in research on mathematical thinking. In Herbert P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 7-47). New York: Academic.

Grouws, Douglas A., & Meier, Sherry L. (1992). Teaching and assessment relationships in mathematics instruction. In Gilah Leder (Ed.), *Assessment and learning of mathematics* (pp. 83-106). Hawthorn: The Austrailian Council for Educational Research.

Gupta, Rajinder M., & Coxhead, Peter (Eds.). (1988). *Cultural diversity and learning efficiency: Recent developments in assessment*. New York: St. Martin's Press.

Hughes, Martin. (1986). *Children and number*. Oxford: Blackwell.

Hunting, Robert P., Davis, Gary E., & Pearn, Catherine A. (1996). Engaging whole number knowledge for rational number learning using a computer-based tool. *Journal for Research in Mathematics Education, 27*, 354-379.

Hunting, Robert P., & Doig, Brian A. (1992). The development of a clinical tool for initial assessment of a student's mathematics learning. In John Izard & Max Stephens (Eds.), *Reshaping assessment practices: Assessment in the mathematical sciences under challenge* (pp. 201-217). Hawthorn: ACER.

Hunting, Robert P. & Doig, Brian A. (1997). Clinical assessment in mathematics: Learning the craft. *Focus on Learning Problems in Mathematics, 19*(3), 29-48.

Hunting, Robert P., Doig, Brian A., & Gibson, Sandra J. (1993). *Initial clinical assessment procedure, Mathematics - Level B (Upper primary).*

Bundoora: The Clinical Mathematics Laboratory, La Trobe University.

Hunting, Robert P., Doig, Brian A., & Nugent, Eroia (1995). *Initial clinical assessment procedure, Mathematics - Level B (Upper primary),* Short version. Bundoora: The Clinical Mathematics Laboratory, La Trobe University.

Hunting, Robert P., Gibson, Sandra J., Pearn, Catherine A., & Doig Brian A. (1995). *Research rationales and technical notes for ICAPM Level B tasks.* Clinical Mathematics Laboratory, La Trobe University.

Hunting, Robert P. & Lamon, Susan. (1995). A re-examination of the role of instructional materials in mathematics education. *Nordisk Matematik Didaktik, (Nordic Studies in Mathematics Education) 3,* 45-64.

Izard, John & Stephens, Max (Eds.) (1992). *Reshaping assessment practices: Assessment in the mathematical sciences under challenge.* Hawthorn: ACER.

Labinowicz, Ed. (1985). *Learning from children.* Menlo Park, CA: Addison Wesley.

Leder, Gilah (Ed.) (1992). *Assessment and learning of mathematics.* Hawthorn: ACER.

Lidz, Carol S. (1987). *Dynamic assessment.* New York: The Guilford Press.

Lidz, Carol S. (1991). *Practitioner's guide to dynamic assessment.* New York: The Guilford Press.

National Council of Teachers of Mathematics (NCTM) (1989). *Curriculum and evaluation standards for school mathematics.* Reston VA: Author.

National Council of Teachers of Mathematics (NCTM) (1995). *Assessment standards for school mathematics.* Reston, VA: Author.

Niss, Mögens (Ed.) (1994). *Investigations into assessment in mathematics education.* Dortrecht: Kluwer.

Niss, Mögens (Ed.) (1994). *Cases of assessment in mathematics education.* Dortrecht: Kluwer.

Pearn, Catherine A., Doig, Brian A., & Hunting, Robert P. (1996). Unmasking "good" mathematicians' mathematical problems: A case study with division. In Helen Forgasz, Tony Jones, Gilah Leder, Julianne Lynch, Kevin Maguire, & Catherine Pearn, (Eds.), *Mathematics: Making connections.* (pp. 221-226). Brunswick: Mathematical Association of Victoria.

Pearn, Catherine A., Merrifield, Marguerite, Mihalic, Helen, & Hunting, Robert P. (1994). *Initial clinical assessment procedure, Mathematics - Level A A (Years 1 & 2).* Bundoora: The Clinical Mathematics Laboratory, La Trobe University.

Pitkethly, Anne & Hunting, Robert P. (1996). A review of research in the area of initial fraction concepts. *Educational Studies in Mathematics, 30,* 5-38.

Romberg, Thomas A. (Ed.) (1992). *Mathematics assessment and evaluation: Imperatives for mathematics educators.* New York: State University of New York Press.

Ludlow, Adalira. (1994). Michael's fraction schemes. *Journal for Research in Mathematics Education, 25,* 50-85.

Schon, Donald A. (1987). *Educating the reflective practitioner.* San Francisco, CA: Jossey-Bass.

Simon, Martin A. & Blume, Glendon W. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal for Research in Mathematics Education, 25,* 472-494.

Steffe, Leslie P. (1990). Mathematics curriculum design: A constructivist perspective. In Leslie P. Steffe & Terry Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 389-398). Hillsdale, NJ: Erlbaum.

Steffe, Leslie P. (1995). Alternative epistemologies: An educator's perspective. In Leslie P. Steffe & Jerry Gale (Eds.), *Constructivism in education* (pp. 489-523). Hillsdale, NJ: Erlbaum.

Steffe, Leslie P. & Cobb, Paul (1988). *Construction of arithmetical meanings and strategies.* New York: Springer-Verlag.

Thompson, Patrick W. & Thompson, Alba G. (1994). Talking about rates conceptually, Part I: A teacher's struggle. *Journal for Research in Mathematics Education, 25,* 279-303.

von Glasersfeld, Ernst (1995). Sensory experience, abstraction, and teaching. In Leslie P. Steffe & Jerry Gale (Eds.), *Constructivism in education* (pp. 369-383). Hillsdale, NJ: Erlbaum.

von Glasersfeld, Ernst (1995). *Radical constructivism: A way of knowing and learning.* London: Falmer Press.

Webb, Norman L. (Ed.). (1993). *Assessment in the mathematics classroom.* Reston, VA: National Council of Teachers of Mathematics.