Action   **Reason**
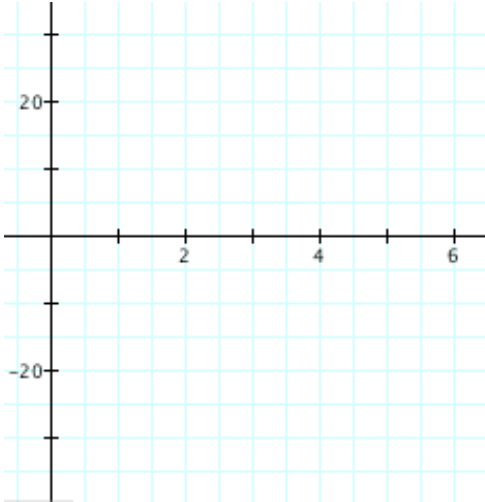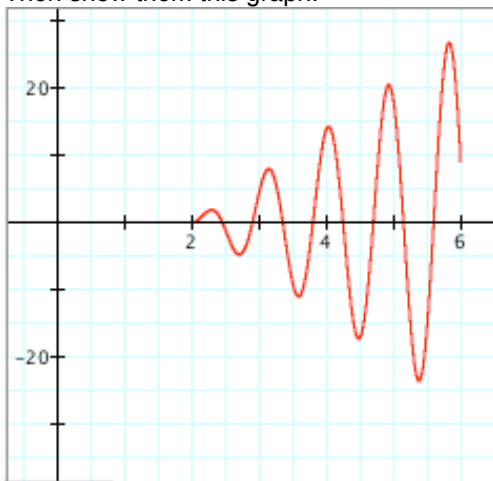
- Odds & Ends

  - The nature of explanations

    - Ask teachers to sketch a graph, using the blank provided here, that has the following property: As *x* varies from 2 to 6, f(x) varies from 0 to 7.5.

      Several teachers described cos(asin(bx)) as varying from cos(-a) to cos(a) since asin(bx) varies from -a to a. In principle, this omits all behavior between -a and a, and it leaves x out of the picture. ALSO NOTE: cos(-a)=cos(a)

      Then show them this graph.

    - Use of landmarks: Break up the function into behavior over "explainable intervals" with respect to its argument. If the argument is a function, then break IT up into explainable intervals determined by the first level of intervals. Etc.

      Analyzing the behavior of a function is a dialectic between looking at the function from the outside-in and from the inside-out. Outside-in considerations take the function as a whole to find intervals of interesting behavior. Inside-out considerations give explanations of behavior over those intervals.

  - Explaining a function's graph

    - A function's graph appears as it does for TWO reasons:

      - The function's definition

      - The conventions of the coordinate system
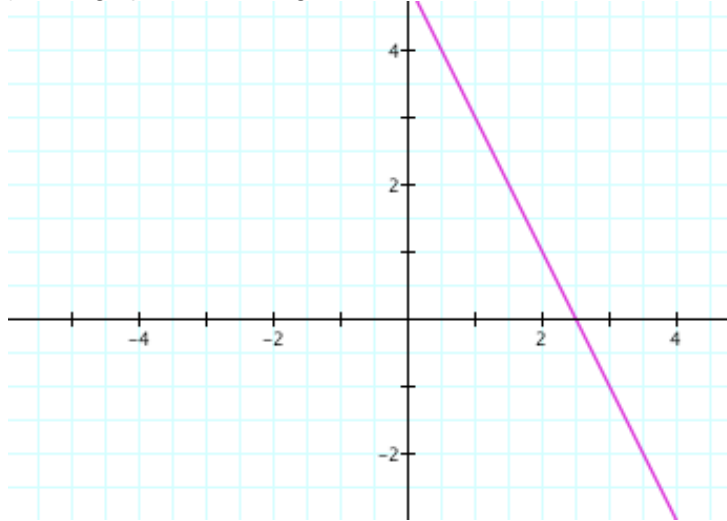
    - The two reasons interact! Example:

      y=5-2x graphed in rectangular coordinates   y=5-2x in polar coordinates

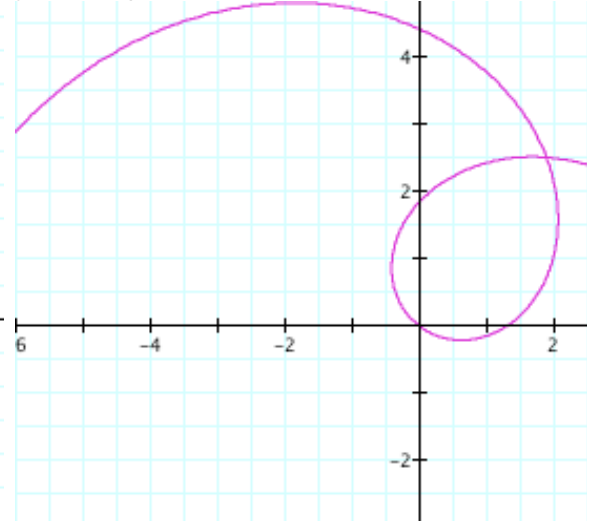| Action	Reason

- The two reasons interact! Example:

y=5-2x graphed in rectangular coordinates

y=5-2x in polar coordinates

- Moral:

  - Explanations MUST draw on meanings, conventions, and context.

    When it is questionable that meanings are shared, USE MEANINGS EXPLICITLY

  - Good explanations provide insight!

    Good explanations help learners (a) understand context, (b) draw connections, (c) rely on meanings

  - If you have trouble explaining something, then you probably aren't using meanings or haven't firmly established meanings.

    The same applies when your students have difficulty giving explanations or understanding a problem.

  - Examine f(theta)=(cos(2theta))/sin(theta). Where does the horizontal line come from?

- Pedagogical comment: I designed this part of the lesson by listening to teachers during the prior class and by looking at the work that they submitted. In both cases I thought about the ROOTS of their difficulties.

- Ask who feels comfortable with the assignment and who struggled with the assignment.

  - Break into groups. Have those who are comfortable mentor those who feel they are struggling. Have them work this problem.
    2. Group Activity

  - After sketching the graph, consider the root difficulties in understanding, making, and interpreting graphs in polar coordinates

    - What is difficult about understanding them?

    - Considerations when teaching polar coordinates

    - Implications of these considerations ffor other areas of mathematics that you will teach

  - Report from Groups

- Review video #3 of Over & Back. What are the central ideas? What are the important pedagogical moves?

- Cities A & B

Action	**Reason**
• Cities A & B	