Exposure Times at Different Focal Ratios (F-Stops)

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Not directly related to math ed research, but potentially a setting in which research might be conducted regarding quantitative reasoning. It has to do with astrophotography and light.

The motive for this example is that I am moving from a telescope with an f-ratio (quotient of focal length and aperture (width of primary lens)) of f/4.5 to a telescope with an f-ratio of f/1.9. I needed to figure out how long my exposures on the f/1.9 telescope need to be to match the 10-minute exposures I'd been taking with my f/4.5 telescope. As you see below, I need exposures of less than 2 minutes on the f/1.9 scope to match the 10-minute exposures on my f/4.5 scope.

The key idea behind everything below is that for a given focal length and aperture, to double the amount of light reaching the camera sensor you need to make the aperture $\sqrt{2}$ times as large, and that if you double the amount of light reaching the sensor, you need only 1/2 the amount of time to reach the same exposure. An interesting aspect of this model is that the unit of time does not matter.

The very first function is the most daunting conceptually. What it does is find the number of factors of $\sqrt{2}$ needed to turn F2 into F1. The function gives the log of the quotient base $\sqrt{2}$. Emily Kuper's dissertation at ASU (under the direction of Marilyn Carlson) does a fantastic job of investigating students' conceptualization of logarithms as finding the number of factors of the base contained in the log's argument. This article in JMB is based on Emily's dissertation.

NB: f-ratio and f-stop are synonymous.

⬇FStops(F1,F2) gives the number of F-stops between an F-stop of F2 and an F-stop of F1.

⊠ $$\mathrm{FStops}(F_1, F_2) = \frac{\ln\left(\frac{F_1}{F_2}\right)}{\ln(\sqrt{2})}$$

⊠ FStops(22, 15) = 1.10508

⊠ FStops(4.5, 1.9) = 2.48785

⊠ FStops(1.9, 4.5) = -2.48785

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⬇LIGHT(F1,F2) gives the number of times as great that the amount of light registered by an F-stop of F2 is than an F-stop of F1..

⊠ $$\mathrm{Light}(F_1, F_2) = 2^{\mathrm{FStops}(F_1, F_2)}$$

⊠ Light(10, 1.9) = 27.7008

⊠ Light(4.5, 1.9) = 5.60942

⊠ Light(1.9, 4.5) = 0.178272

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⬇NewTime(OldTime,F1,F2) gives the time at F-stop F2 to have the same exposure as as an exposure at F-Stop F1 for OldTIme minutes (or seconds).

⊠ $$\mathrm{NewTime}(\mathrm{OldTime}, F_1, F_2) = \frac{\mathrm{OldTime}}{\mathrm{Light}(F_1, F_2)}$$

⊠ NewTime(600, 4.5, 1.9) = 106.963

⊠ NewTime(NewTime(600, 4.5, 1.9), 1.9, 4.5) = 600

⊠ NewTime(10, 4.5, 1.9) = 1.78272

⊠ 60NewTime(10, 4.5, 1.9) = 106.963