Musing on musings

Shlomo Vinner (2018) *Mathematics, Education, and Other Endangered Species: From Intuition to Inhibition.* xx+141pp. Springer. ISBN 978-3-319-90034-6
$65.00

Reviewed by
Patrick W. Thompson
Arizona State University

Few people have contributed to mathematics education as much as Shlomo Vinner over the past 45 years. Mathematics education graduate students learn of Vinner's landmark ideas of concept image and concept definition (Tall & Vinner, 1981; Vinner, 1983, 1987; Vinner & Dreyfus, 1989) and pseudo-conceptual and -analytical thought processes (Vinner, 1997). Also, researchers studying students' understandings of function often start with Vinner's publications (Vinner, 1983, 1992, 1987; Vinner & Dreyfus, 1989). These publications revealed problematic mathematical understandings and ways of thinking among students. They also hid a somewhat dystopian view of mathematics education in schools and colleges. Vinner's book puts this view in full display.

The volume does not contain a synthesis of research. Rather, it is an introspective book, disconnected from other points of view. It exemplifies Vinner's stance as stated in (Vinner, 2011): Wittgenstein said he had no interest in seeing how his thoughts overlapped with those of other philosophers, and Vinner followed by saying, "I consider [Wittgenstein's statement] as a call for other authors to follow him, and not to follow always the form of common papers of which he, undoubtedly, was quite aware." (Vinner, 2011, pp. 247–248). Vinner makes good on his stance in this book. You will find few connections to other research in mathematics education.

The book has a definite stream-of-consciousness feel. It includes 17 chapters without any visible organization. Chapters 1-3 are loosely related to Vinner's view of mathematics and the role of mathematics in personal and social life. Chapters 4-6 address a variety of topics with the thread of pseudo-conceptual and pseudo-analytical behaviors tying them together. Chapter 7 presents Vinner's negative views on e-learning. Chapters 8-11 begin with a focus on the nature of mathematics, with side trips into the role of examples, the importance of imagination, and Vinner's thoughts on infinity. Chapter 12's title suggests it is about values in education, but a major focus is on the value of procedures. Chapter 13 presents Vinner's view on the preparation of elementary mathematics teachers, Chapter 14 is about proofs by contradiction, Chapter 15 presents Bob Davis' notion of visually moderated sequences (Davis, 1984; Davis et al., 1978) and their connection to students' pseudo-analytic and pseudo-conceptual behaviors. Chapter 16 discusses science versus common sense and theology, and Chapter 17 discusses rationality being at the core of mathematics and science.

Stream of consciousness writing is not without merits. It allows a writer to draw connections shrewdly that would otherwise require tomes of academic prose. This is illustrated nicely in

Vinner's unorthodox, and sometimes illuminating, connections among mathematics, education, religion, and even attitudes toward death.

Stream of consciousness writing also has drawbacks. It can allow contradictory thoughts to creep in depending on one's moods at different times of writing. For example, Vinner sometimes speaks of students' proclivity to engage in pseudo-conceptual or pseudo-analytic behaviors as if they are trying to dupe their teachers. At other times, he speaks of these same behaviors as arising quite understandably from students' experience of common modes of instruction in which these very behaviors seem to be expected of them. In the same vein, he points sometimes to students' disinterest in engaging intellectually with tasks as a root of pseudo-behaviors and at other times speaks of the human spirit as being naturally inquisitive.

Chapter 8, *What is Mathematics?*, is particularly disappointing in light of what I believed Vinner actually could offer. According to Vinner, "Mathematics is a collection of mathematical theories. Each theory is a set of theorems and has models in which the theorems hold. There is also a deductive system for proving theorems" (p. 63). This is followed by examples of proving algebraic identities and a long discussion of proof by induction. I am mystified by how Vinner thought the content of Chapter 8 possibly responds to the question, "What is mathematics?" I was hoping for something more along the lines of Harel's (2008) penetrating discussion of mathematics as a human activity.

Springer bears some responsibility for the book's final form. The number of one-sentence paragraphs is remarkable. Also, I had not seen a one-page chapter until finding one here. The book contains 132 pages; 11 of 17 chapters are four pages or less. It is understandable the book fails to penetrate many ideas Vinner raises when devoting less than four pages to them.

An aspect of Vinner's book I greatly appreciated did not merit a chapter of its own, yet is at the forefront in many passages. It is Vinner's repeated connections between thinking mathematically and behaving morally. Mathematics, he claims, is about reasoning carefully and non-judgmentally. It is an attitude toward life Vinner argues would serve everyone well.

References

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Greenwood Publishing Group.

Davis, R. B., Jockusch, E., & McKnight, C. (1978). Cognitive processes involved in learning algebra. *Journal of Children's Mathematical Behavior*, *2*(1), 10–320.

Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics From the Perspective of Mathematicians* (pp. 265–290). Mathematical Association of America.

Tall, D. O., & Vinner, S. (1981). Concept images and concept definitions in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151–169.

Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, *14*(3), 293–305. https://doi.org/10.1080/0020739830140305

Vinner, S. (1992). The function concept as a prototype for problems in mathematics learning. In G. Harel & E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 195–214). Mathematical Association of America.

Vinner, S. (1997). Pseudo-conceptual and pseudo-analytic thought processes in mathematics learning. *Educational Studies in Mathematics*, *34*, 97–129.

Vinner, S. (2011). The role of examples in the learning of mathematics and in everyday thought processes. *ZDM*, *43*(2), 247–256. https://doi.org/10.1007/s11858-010-0304-3

Vinner, S. (1987). Continuous functions: Images and reasoning in college students. In J. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education* (pp. 177–183). Université de Montréal.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education*, *20*(4), 356–366.