RUME 1
Theorizing Precisely

Pat Thompson
Week 8
October 16, 2018

Students' Issues with Uses of "Divide"

"Divide **5** into **3**"

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\ 3}$

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\ 3}$

"Divide 5 into 3 pieces"

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\ 3}$

"Divide 5 into 3 pieces"

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

"Divide 5 into 3 pieces"

$5\overline{)\ 3}$

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\ 3}$

"Divide 5 into 3 pieces"

$3\overline{)\ 5}$

??

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\ 3}$

"Divide 5 into 3 pieces"

$3\overline{)\ 5}$

??

"Cut up 5 into 3 pieces"

5

_____

_____ 1/3 of 5

_____ 1/3 of 5

_____ 1/3 of 5

Students' Issues with Uses of "Divide"

"Divide 5 into 3"

$5\overline{)\,3}$

Numerical operation

"Divide 5 into 3 pieces"

$3\overline{)\,5}$

??

Unclear — numerical operation or quantitative operation?

"Cut up 5 into 3 pieces"

5

1/3 of 5

1/3 of 5

1/3 of 5

Quantitative operation

Four Students' Meanings of "Change"

"x changed from 2 to 5"

x

Four Students' Meanings of "Change"

"x changed from 2 to 5"

2

Four Students' Meanings of "Change"

"x changed from 2 to 5"

5

Four Students' Meanings of "Change"

"x changed from 2 to 5"

5 $x = 2$

Four Students' Meanings of "Change"

"x changed from 2 to 5"

5 $x = 5$

Four Students' Meanings of "Change"

"x changed from 2 to 5"

5

$x = 5$

$2 \xrightarrow{x} 5$

Four Students' Meanings of "Change"

"x changed from 2 to 5"

5

$x = 5$

$2 \xrightarrow{x} 5$

$2 \xrightarrow{x} 5$

Four Students' Meanings of "Change"

"x changed from 2 to 5"

	5	$x = 5$	$2 \xrightarrow{x} 5$	$2 \xrightarrow{x} 5$
What to call it?	?	Become something different	Completed variation	Variation in progress

Quantity

Terms commonly thought of as indicating a quantity

Length	Object? Attribute? Unit
Area	Object? Attribute? Unit
Volume	Object? Attribute? Unit

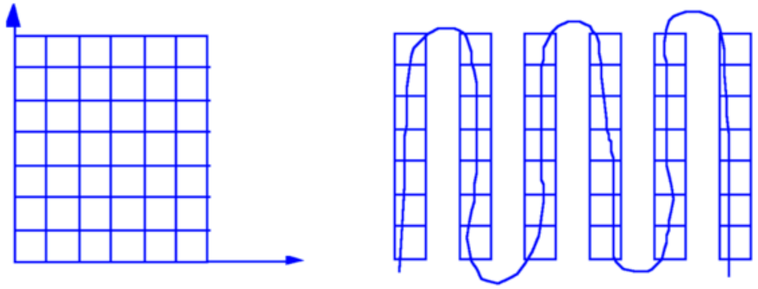
Quantity

Terms commonly thought of as indicating a quantity

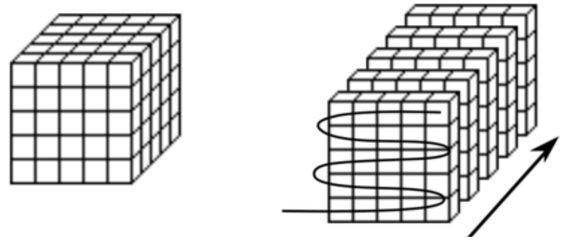
Length Object? Attribute? Unit

Area Object? Attribute? Unit

Volume Object? Attribute? Unit



Area as a One Dimensional Quantity



Volume as a One Dimensional Quantity

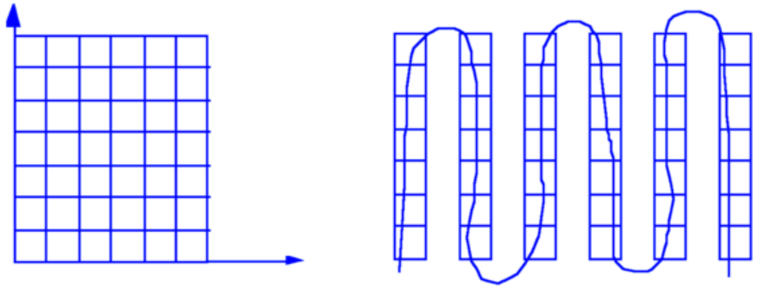
Quantity

Terms commonly thought of as indicating a quantity

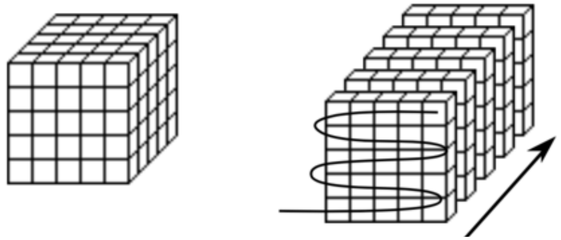
Length Object? Attribute? Unit

Area Object? Attribute? Unit

Volume Object? Attribute? Unit



Area as a One Dimensional Quantity



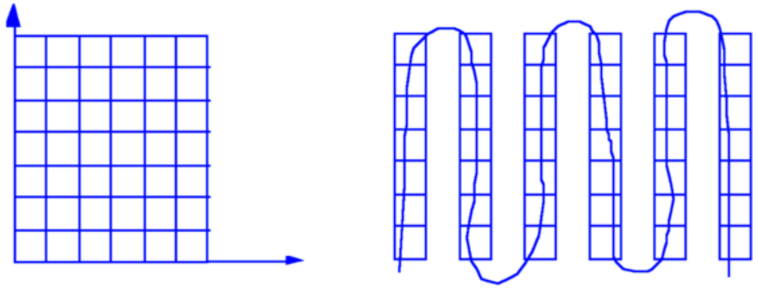
Volume as a One Dimensional Quantity

A quantity is *conceived* by *someone*.
It is *essential* you ask, "What is the object, the attribute, the quantification *this* student (teacher, person) has conceived?"

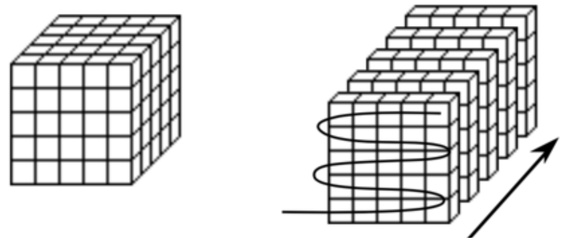
Quantity

Terms commonly thought of as indicating a quantity

Length	Object? Attribute? Unit
Area	Object? Attribute? Unit
Volume	Object? Attribute? Unit
Angle Measure	Object? Attribute? Unit
Profit	Object? Attribute? Unit
Economy	Object? Attribute? Unit
Fuel efficiency	Object? Attribute? Unit
Relative size	Object? Attribute? Unit
Mass	Object? Attribute? Unit
Force	Object? Attribute? Unit
Torque	Object? Attribute? Unit
Energy	Object? Attribute? Unit



Area as a One Dimensional Quantity



Volume as a One Dimensional Quantity

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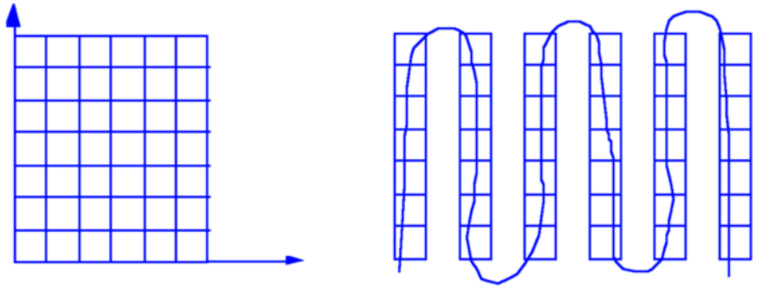
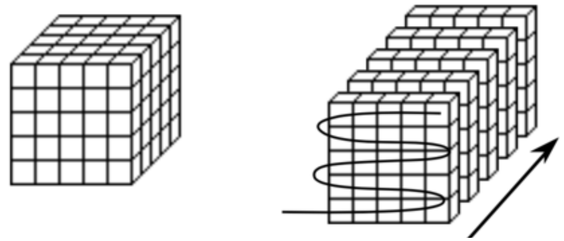


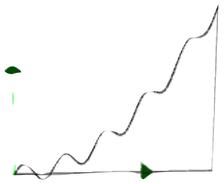
Area as a One Dimensional Quantity



Volume as a One Dimensional Quantity

- **Magnitude (student measured his height in inches and cm)**
  - How tall are you in inches? 79
  - How tall are you in centimeters? About 201
  - How many centimeters is 79 inches? Huh?

Covariation — Experiencing the Struggle



Rotations about line parallel to *x*, *y*, or *z* axis passing through Center

$$R_x(X, \theta, \text{Center}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} (X - \text{Center}) + \text{Center}$$

$$R_y(X, \theta, \text{Center}) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} (X - \text{Center}) + \text{Center}$$

$$R_z(X, \theta, \text{Center}) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (X - \text{Center}) + \text{Center}$$

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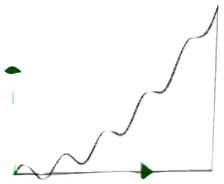
Covariation — Experiencing the Struggle



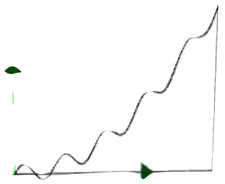
$d = \text{slider}(0.005, 0.1)$ d = 0.013075

$n = \text{clamp}(m, 0, 2\pi)$

$f(x) = \frac{x^2}{3} + 0.2 \sin(10x)$

$C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Covariation — Experiencing the Struggle



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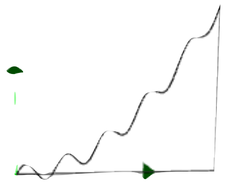
What do these statements do and how do they do it?

- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_y\left(\begin{bmatrix} \pi t \\ f(\pi t) \\ 0 \end{bmatrix}, n, C\right), \text{radius} = d$
- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_y\left(\begin{bmatrix} \pi \\ tf(\pi) \\ 0 \end{bmatrix}, n, C\right), \text{radius} = d$
- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_y\left(\begin{bmatrix} t\pi \\ 0 \\ 0 \end{bmatrix}, n, C\right), \text{radius} = d$

Covariation — Experiencing the Struggle



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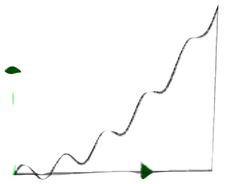

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$f(x) = \frac{x^2}{3} + 0.2 \sin(10x)$

$C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

What does this statement do and how does it do it?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(R_y\left(\begin{bmatrix} u\pi \\ f(u\pi) \\ 0 \end{bmatrix}, vn, C \right) \text{ if Surface} = 1 \right)$$

What does these statement do and how do they do it?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(R_y\left(\begin{bmatrix} u\pi \\ 0 \\ 0 \end{bmatrix}, vn, C \right) \text{ if Surface} = 1 \right)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(R_y\left(\begin{bmatrix} \pi \\ uf(\pi) \\ 0 \end{bmatrix}, vn, C \right) \text{ if Surface} = 1 \right)$$

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