PART I

Foundations

1

PUTTING PHILOSOPHY TO WORK

Coping with Multiple Theoretical Perspectives

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In inviting me to write this chapter on philosophical issues in mathematics education, the editor has given me the leeway to present a personal perspective rather than to develop a comprehensive overview of currently influential philosophical positions as they relate to mathematics education. I invoke this privilege by taking as my primary focus an issue that has been the subject of considerable debate in both mathematics education and the broader educational research community, that of coping with multiple and frequently conflicting theoretical perspectives. The theoretical perspectives currently on offer include radical constructivism, sociocultural theory, symbolic interactionism, distributed cognition, information-processing psychology, situated cognition, critical theory, critical race theory, and discourse theory. To add to the mix, experimental psychology has emerged with a renewed vigor in the last few years. Proponents of various perspectives frequently advocate their viewpoint with what can only be described as ideological fervor, generating more heat than light in the process. In the face of this sometimes bewildering array of theoretical alternatives, the issue I seek to address in this chapter is that of how we might make and justify our decision to adopt one theoretical perspective rather than another. In doing so, I put philosophy to work by drawing on the analyses of a number of thinkers who have grappled with the thorny problem of making reasoned decisions about competing theoretical perspectives.

In the first section of the chapter, I follow Schön (1983) in challenging what he termed the positivist epistemology of practice wherein practical reason is construed as the application of theory. My goal in doing so is to question the repeated attempts that have been made in mathematics education to derive instructional prescriptions directly from background theoretical perspectives. I argue that it is instead more productive to compare and contrast various perspectives by using as a criterion the manner in which they orient and constrain the types of questions that are asked about the learning and teaching of mathematics, the nature of the phenomena that are investigated, and the forms of knowledge that are produced.

In the second section of the chapter, I argue that mathematics education can be productively construed as a design science, the collective mission of which involves developing, testing, and revising conjectured designs for supporting envisioned learning processes. As will become apparent, my intent in developing this viewpoint is inclusive rather than exclusive. I therefore construe designs broadly and clarify that the learning processes might, for example, be those of individual students or individual teachers, classroom communities or professional teaching communities, or indeed of schools or school districts as organizations. This perspective on our collective activity as mathematics educators gives rise to a second criterion for comparing and contrasting background theoretical positions, namely that of how they might contribute to the enterprise of formulating, testing, and revising conjectured designs for supporting mathematical learning. This second criterion is therefore concerned with the nature of the potentially useful work that different theoretical positions might do.

In section three, I draw both on the work of John Dewey and on the more recent contributions of neo-Pragmatist philosophers to question the relevance of a number of philosophical distinctions that have featured prominently in the mathematics education research literature. Foremost among these is the central problem of traditional epistemology, that of the opposition between philosophical realism and constructivist positions that deny that ontological reality is knowable. In doing so, I outline an alternative position that Putnam (1987) terms pragmatic realism and clarify its relevance to the issue of comparing and contrasting different theoretical perspectives in mathematics education research. Against this background in section four and five, I then sharpen the two criteria that I propose for comparing background theoretical positions. The first criterion concerns the nature of the phenomena that are investigated and is often framed in terms of whether a particular perspective treats activity as being primarily individual or social in character. I argue that this dichotomy is misleading in that it assumes that what is meant by *the individual* is self-evident and theory neutral. As an alternative, I propose to compare and contrast different theoretical positions in terms of how they characterize individuals, be they students, teachers, or administrators. The second criterion concerns the potentially useful work that different theoretical positions might do and here I differentiate Dewey's sophisticated account of pragmatic justification and his related analysis of verification and truth from a purely instrumental approach in which methods or strategies that enable us to reach our goals are deemed to be true, and those that do not are treated as false.

In the subsequent sections of the chapter, I use these two criteria as my primary points of reference as I discuss four broad theoretical positions: experimental psychology, cognitive psychology, sociocultural theory, and distributed cognition. Experimental psychology refers to the psychological research tradition in which the primary methods employed involve experimental and quasi-experimental designs, preferably with the random assignment of subjects. The cognitive psychology tradition on which I focus involves accounting for teachers' and students' inferred interpretations and understandings in terms of internal cognitive structures and processes. Sociocultural theory and distributed cognition provide points of contrast with this psychological tradition in that people's activity and learning are considered to be situated with respect to the social and cultural practices in which they participate. As I clarify, sociocultural theory developed largely independently of western cognitive psychology by drawing directly on the writings of Vygotsky and Leont'ev and is concerned with people's induction into and participation in relatively broad cultural practices. In contrast, distributed cognition emerged in reaction to mainstream cognitive psychology and tends to focus on people's activity in their immediate physical, symbolic, and social environments.

I conclude from the comparison of the four perspectives that each has limitations in terms of the extent to which it can contribute to the enterprise of formulating, testing, and revising designs for supporting learning. As part of the analysis, I outline the historical origins of each perspective in order to understand adequately the types of phenomena that are investigated and the forms of knowledge produced. I note that the concerns and interests that motivated the development of these four perspectives differ from those of mathematics educators. In light of this conclusion, I propose and illustrate how we can view the various theoretical perspectives as sources of ideas to be appropriated and adapted to our purposes as mathematics educators. In the final section of the chapter, I then step back to consider the issue of competing, incommensurable theories more broadly. In doing so, I locate the approach I have taken within a philosophical tradition that seeks to transcend the longstanding dichotomy between the quest for a neutral framework for comparing theoretical perspectives on the one hand and the view that we cannot reasonably compare perspectives on the other hand.

THE POSITIVIST EPISTEMOLOGY OF PRACTICE

In his influential book *The Reflective Practitioner*, Donald Schön (1983) explicitly targets what he terms the positivist epistemology of practice wherein practical reasoning is accounted for in terms of the application of abstract theoretical principles to specific cases. As Schön (1983, 1986, 1987) notes, this epistemology is apparent in teacher education programs that (a) initially emphasize the acquisition of theoretical principles derived from fields such as psychology and sociology, (b) then focus on the process of applying these principles in subject matter methods courses before (c) allowing future teachers to engage in this application process in field experiences. This epistemology is also implicit in instructional approaches that separate students' initial acquisition of mathematical concepts, strategies, or procedures from their subsequent application of those concepts and methods. In the research literature, the positivist epistemology of practice is central to what De Corte, Greer, and Verschaffel (1996) term the first wave of the cognitive revolution, the goal of which was to formulate internal information-processing mechanisms that account for observed relations between the external stimulus environment and documented responses. Anderson, Reder, and Simon (1997) exemplified this position in an exchange with Greeno (1997) when they argued that the contributions of cognitive analyses of this type stem from the analytic power they provide for extracting principles that can generalize from one setting to another. Anderson et al. suggested that teachers should base their instruction on these principles while relying on common sense and professional experience when issues arise in the classroom that psychologists cannot yet answer. Most importantly for the purposes of this chapter, the positivist epistemology of practice is also apparent in debates about the implications of background theoretical positions for mathematics learning and teaching.

The most prominent case in which attempts have been made to derive instructional implications directly from a background theory is that of the development of the general pedagogical approach known as *constructivist teaching*. This pedagogy claims to translate the theoretical contention that learning is a constructive activity directly into instructional recommendations. As Noddings (1990) and Ball and Chazan (1994) observe, it is closely associated with the dubious assertion that "telling is bad" because it deprives students of the opportunity to construct knowledge for themselves. For his part, J. P. Smith (1996) clarifies that adherents of the pedagogy tend to frame teachers' proactive efforts to support their students' learning as interfering with students' attempts to construct meaning for themselves. As Smith demonstrates, in emphasizing what the teacher does not do compared with traditional instructional practices, the teacher's role in supporting learning is cast in relatively passive terms, thereby resulting in a sense of loss of efficacy.

It is important to note that constructivism does not have a monopoly on questionable reasoning of this type in which background theoretical contentions that are *descriptive* in nature are translated directly into instructional maxims that are *prescriptive*. For example, an advocate of symbolic interactionism might argue that as learning involves the negotiation of meaning, students should be encouraged to continually discuss their differing interpretations. Similarly, a devotee of the distributed view of intelligence might argue that as cognition is stretched over individuals, tools and social contexts, it is important to ensure that students' mathematical activity involves the use of computers and other tools. In each of these cases, the difficulty is not with the background theory but with the relation that is assumed to hold between theory and instructional practice. As I have argued elsewhere (P. Cobb, 2002), pedagogical proposals developed in this manner involve a category error wherein the central tenets of a descriptive theoretical perspective are transformed directly into instructional prescriptions. It is noteworthy that similar instructional recommendations (e.g., the importance of small group work) are frequently derived from contrasting theoretical positions (e.g., constructivism, symbolic interactionism, and distributed intelligence). The resulting pedagogies are underspecified and are based on ideology (in the disparaging sense of the term) rather than empirical analyses of the process of students' learning and the means of supporting it in specific domains.

Later in this chapter, I will build on this critique of the positivist epistemology of practice by proposing an alternative view of the relation between theory and practice. In doing so, I will question the assumption that theory consists of decontextualized propositions, statements, or assertions that are elevated above and stand apart from the activities of practitioners. For the present, it suffices to note that work in the philosophy, sociology, and history of science, sparked by the publication of Thomas Kuhn's (1962) landmark book, *The Structure of Scientific Revolutions*, has challenged this view of theoretical reasoning in the natural sciences. In this and subsequent publications (Kuhn, 1970, 1977), Kuhn developed an analysis of the both processes by which scientists develop theory within an established research tradition and those by which they choose between competing research traditions. In the case of theory development within an established research tradition, he followed Michael Polanyi (1958) in questioning the assumption that scientists fully explicate the bases of their reasoning by presenting analyses of a number of historical cases. His goal in doing so was to demonstrate that the development and use of theory necessarily involves tacit suppositions and assumptions that scientists learn in the course of their induction into their chosen specialties. Kuhn went on to clarify that these implicit aspects of scientific reasoning serve as a primary means by which scientists distinguish between insiders and outsiders in the process of establishing the boundaries of scientific communities.

Kuhn (1962) extended this argument about the tacit aspects of scientific reasoning when he considered how scientists choose between competing research traditions by arguing that "there is no neutral algorithm of theory-choice, no systematic decision procedure which, properly applied, must lead each individual in the group to the same decision" (p. 200). Not surprisingly, Kuhn was accused of claiming that the process by which scientists resolve disputes is irrational. It is therefore important to stress that he was not in fact questioning the rationality of scientists. Instead, he was challenging the dominant view that scientific reasoning could be modeled as a process of applying general rules and procedures to specific cases. As Bernstein (1983) observed, "Kuhn always intended to distinguish forms of rational persuasion and argumentation that take place in scientific communities from those irrational forms of persuasion that he has been accused of endorsing" (p. 53). In responding to his critics, Kuhn (1970) subsequently clarified that

> what I am denying is neither the existence of good reasons [for choosing one theory over another] nor that these reasons are of the sort usually described. I am, however, insisting that such reasons constitute values to be used in making choices rather than rules of choice. Scientists who share them may nevertheless make different choices in the same concrete situation… [In concrete cases, scientific values such as] simplicity, scope, fruitfulness, and even accuracy can be judged quite differently (which is not to say that they can be judged arbitrarily) by different people. Again, they may differ in their conclusions without violating any accepted rule. (p. 262)

Bernstein (1983) summarized the debate incited by Kuhn's analysis of scientific reasoning by cautioning that

> one must be sensitive to and acknowledge the important differences between the nature of scientific knowledge and other forms of knowledge. But the more closely we examine the nature of this scientific knowledge that has become the paradigm of theoretical knowledge, the more we realize that the character of rationality in the sciences, especially in matters of theory-choice, is closer to those features of rationality that have been characteristic of the tradition of practical philosophy than to many modern images of what is supposed to be the character of genuine *episteme* [i.e., the application of general, decontextualized methods to specific cases]. (p. 47)

In speaking of practical philosophy, Bernstein was referring specifically to the work of Gadamer (1975). The specific phenomenon that Gadamer analyzed when he developed his philosophy of practical activity was the process by which people interpret and understand texts, particularly religious texts. In developing his position, Gadamer responded to earlier work in this area that instantiated the positivist epistemology of practice by differentiating between general methods of interpretation and understanding on the one hand, and the process of applying them to specific texts on the other hand. Gadamer rejected this distinction, arguing that every act of understanding involves interpretation, and all interpretation involves application. On this basis, he concluded that the characterization of theoretical reasoning as the application of general, decontextualized methods serves both to mystify science and to degrade practical reasoning to technical control.

The philosophical positions that Kuhn, Gadamer, and others developed to challenge the dominance of positivist epistemology of practice foreshadowed a number of more recent developments that are familiar to mathematics education researchers. For example, their argument that reasoning in all domains, including those that are typically characterized as formal and abstract, has much in common with practical reasoning anticipated the view proposed by a number of educational researchers that people's reasoning is situated with respect to their participation in specific activities or practices (e.g., Beach, 1999; J. S. Brown, Collins, & Duguid, 1989; P. Cobb & Bowers, 1999; Forman, 2003; Greeno & The Middle School Mathematics Through Applications Project Group, 1998; Saxe, 1991; Sfard, 1998). Relatedly, their characterization of people as members of intellectual communities anticipated investigations in which the community of practice has been taken as a primary unit of analysis (e.g., Lave & Wenger, 1991; Rogoff, 1990; Stein, Silver, & Smith, 1998; Wenger, 1998).

Kuhn's and Gadamer's arguments are also directly relevant to the issues that I seek to address in this chapter. For example, they call into question attempts to develop a so-called *philosophy of mathematics education* if the goal is to be normative and prescriptive by providing foundational principles that are intended to guide the activity of mathematics education researchers and practitioners. Instead, their arguments orient us to see research, theorizing, and indeed philosophizing as distinct forms of practice rather than activities whose products provide a viable foundation for the activities of practitioners. It is for this reason that I shy away from the approach of first surveying currently fashionable philosophical positions and then deriving implications for mathematics education research and practice from them. Instead, I take as my starting point several background theoretical positions that are already currently influential in mathematics education research: experimental psychology, cognitive psychology, sociocultural theory, and distributed cognition. At the most immediate level, my goal in discussing these theoretical positions is descriptive rather than prescriptive in that I explicate their basic tenets and consider their potential contributions to mathematics education research. This first goal gives rise to a second that is inherently philosophical in nature, that of moving beyond the realization that there is no neutral algorithm of theory choice by considering how we might sensibly compare and contrast different theoretical perspectives given our concerns and interests as mathematics educators. In my view, this latter (meta)issue is of the greatest importance given the profusion of perspectives that has become emblematic of educational research in general, and of mathematics education research in particular.

In order to address this (meta)issue, it is necessary not merely to be explicit about the criteria of comparison, but to justify them. My goal in doing so is not to formulate a general method or procedure for choosing between these and other theoretical perspectives, but to initiate a conversation in which we, as mathematics education researchers, can begin to work through the challenges posed by a proliferation of perspectives. As the neo-pragmatist philosopher Rorty (1979) clarified in his much cited book, *Philosophy and the Mirror of Nature*, philosophy so construed is therapeutic in that it does not presume to tell people how they should act in particular types of situations. Instead, its goal is to enable people themselves to cope with the complexities, tensions, and ambiguities that characterize the settings in which they act and interact. As will become apparent, such an approach seeks to avoid both the unbridled relativism evident in the contention that one cannot sensibly weigh the potential contributions of different theoretical perspectives and the unhealthy fanaticism inherent in the all-too-common claim that one particular perspective gets the world of teaching and learning right.

As an initial criterion, I propose comparing and contrasting the different theoretical perspectives in terms of the manner in which they orient and constrain the types of questions that are asked about the learning and teaching of mathematics, and thus the nature of the phenomena that are investigated and the forms of knowledge produced. Adherents to a research tradition view themselves as making progress to the extent that they are able to address questions that they judge to be important. The content of a research tradition, including the types of phenomena that are considered to be as significant, therefore represent solutions to previously posed questions. However, as Jardine (1991) demonstrates by means of historical examples, the questions that are posed within one research tradition frequently seem unreasonable and, at times, unintelligible from the perspective of another tradition. In Lakatos' (1970) formulation, a research tradition comprises positive and negative heuristics that constrain (but do not predetermine) the types of questions that can be asked and those that cannot, and thus the overall direction of the research agenda. In my view, delineating the types of phenomena that can be investigated within different research traditions is a key step in the process of comparing and contrasting them. As Hacking (2000) observed, while the specific questions posed and the ways of addressing them are visible to researchers working within a given research tradition, the constraints on what is thinkable and possible are typically invisible. This initial criterion is therapeutic in Rorty's (1979) sense in that the process of comparing and contrasting perspectives provides a means both of deepening our understanding of the research traditions in which we work, and of enabling us to de-center and develop a basis for communication with colleagues whose work is grounded in different research traditions.

MATHEMATICS EDUCATION AS A DESIGN SCIENCE

The second criterion that I propose for comparing different theoretical perspectives focuses on their potential usefulness given our concerns and interests as mathematics educators. My treatment of this criterion is premised on the argument that mathematics education can be productively viewed as a design science, the collective mission of which involves developing, testing, and revising conjectured designs for supporting envisioned learning processes. As an illustration of this collective mission, the first two *Standards* documents developed by the National Council of Teachers of Mathematics (1989, 1991) can be viewed as specifying an initial design for the reform of mathematics teaching and learning in North America. The more recent *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) propose a revised design that was formulated in response to developments in the field, many of which were related to attempts to realize the initial design. Significantly, as part of this revision process, the designers found it necessary to substantiate their proposals by developing a research companion volume (Kilpatrick, Martin, & Schifter, 2003). As a second illustration, the functions of leadership in mathematics education at the level of schools and school districts include mobilizing teachers and other stakeholders to notice, face, and take on the task of improving mathematics instruction (Spillane, 2000). Ideally, this task involves the iterative development and revision of designs for improvement as informed by ongoing documentation of teachers' instructional practices and students' learning (Confrey, Castro-Filho, & Wilhelm, 2000; Fishman, Marx, Blumenfeld, & Krajcik, 2004; Hill, 2001; McLaughlin & Mitra, 2004, April). At the classroom level, the design aspect of teaching is particularly evident in Stigler and Hiebert's (1999) description of the process by which Japanese mathematics teachers collaborate to develop and revise the design of lessons. This aspect of teaching is also readily apparent in analyses that researchers such as Ball (1993), Lampert (2001), and Simon (1995) have developed of their own instructional practices.

Although the three illustrations focus on developing, testing, and revising designs at the level of a national educational system, a school or school district, and a classroom respectively, the ultimate goal in each case is to support the improvement of students' mathematical learning. As a point of clarification, I should stress that the contention that mathematics education can be productively viewed as a design science is *not* an argument in favor of one particular research methodology such as design experiments. Rather than being narrowly methodological, the claim focuses on our collective mission and thus on the concerns and interests inherent in our work. This framing of mathematics education research therefore acknowledges that a wide spectrum of research methods ranging from experimental and quasi-experimental designs to surveys and ethnographies can contribute to this collective enterprise. Furthermore, this framing recognizes the value of theoretical analyses that result in the development of conceptual tools that can contribute to this enterprise. The fruitfulness of the framing stems from the manner in which delineates core aspects of mathematics education research as a disciplinary activity. These core aspects include:

- Specifying and clarifying the prospective endpoints of learning
- Formulating and testing conjectures about both the nature of learning processes that aim towards those prospective endpoints and the specific means of supporting their realization (Confrey & Lachance, 2000; Gravemeijer, 1994a; Simon, 1995)

This formulation is intended to be inclusive in that the learning processes of interest could be those of individual students or teachers, of classroom or professional teaching communities, or indeed of schools or school districts viewed as organizations.

A discussion of the relative merits of various goals or envisioned endpoints for students', teachers', and school districts' learning is beyond the scope of this chapter. It is, however, worth noting that the choice of goals and thus of what counts as improvement is not a matter of mere subjective whim or taste. Instead, our choices involve judgments that are eminently discussable in that we are expected to support them by giving reasons. As an illustration, I outline my view of the primary issues that should be considered when formulating endpoints for students' learning. I propose that, at a minimum, the process of formulating instructional goals should involve delineating central mathematical ideas in particular mathematical domains and clarifying more encompassing activities such as mathematical argumentation and modeling (Lehrer & Lesh, 2003). In addition, I contend that it is important to justify the proposed endpoints in terms of the types of future activities to which they might give students access. One important consideration is therefore the extent to which the envisioned forms of mathematical reasoning have what Bruner (1986) termed clout by enabling students to participate in significant out-of-school practices in relatively substantial ways. For example, it is apparent that public policy discourse increasingly involves the formulation and critique of data-based arguments. Students' development of the relatively sophisticated forms of statistical reasoning that are implicated in such arguments therefore has clout in that it enables students to participate in a type of discourse that is central to what Delpit (1988) termed the culture of power (cf. G. W. Cobb, 1997).

In my view, it is also important to take explicit account of the function of schools in differentiating between and sorting students when gauging the extent to which envisioned forms of mathematical reasoning to have clout (cf. Secada, 1995). As an example, Moses and C. Cobb (2001) clarify that the goal of the Algebra Project is to make it possible for *all* students to have access to and to succeed in high school algebra courses that function as gatekeepers to college preparatory tracks, and thus to future educational and economic opportunities. As Moses and Cobb also indicate, cultivating students' development of mathematical interests such that they come to view classroom mathematical activities as worthy of their engagement should be an important goal in its own right. Although many students will not pursue career trajectories beyond school that involve direct engagement in mathematical activity, it is nonetheless reasonable to propose that mathematical literacy include an empathy for and sense of affiliation with mathematics together with the desire and capability to learn more about mathematics when the opportunity arises.

The importance of this latter goal becomes even more apparent when we note that an increasing appreciation for mathematics inherent in the development of mathematical interests is compatible with what D'Amato (1992) refers to as *situational significance* wherein students come to view engagement in mathematical activities as a means of gaining experiences of mastery and accomplishment, and of maintaining valued relationships with teachers and peers. D'Amato contrasts this general way in which learning mathematics in school come to have value for students with a second he terms *structural significance*. In this second case, students come to view achievement in mathematics as a means of attaining other ends such as entry to college and high-status careers, or acceptance and approval in household and other social networks. As he notes, not all students have access to a structural rationale for learning mathematics in school. Gutiérrez (2004) observes, for example, that many urban students do not see themselves going to college, hold activist stances, have more pressing daily concerns (e.g., housing, safety, healthcare), or do not believe that hard work and effort will be rewarded in terms of future educational and economic opportunities. As D'Amato (1992), Erickson (1992), and Mehan, Hubbard, & Villanueva (1994) all document, students' access to a structural rationale varies as a consequence of family history, race or ethnic history, class structure, and caste structure within society. Their analyses indicate that failure to give all students access to a situational rationale for learning mathematics by framing the cultivation of mathematical interests as an explicit goal of design and teaching will result in what Nicholls (1989) termed inequities in motivation.

The issues on which I have focused when discussing the formulation of goals for students' learning clearly reflect my personal view. However, the fact that they are open to critique and challenge indicates that I cannot choose goals on the basis of unarticulated whim or fancy. For example, it might be argued that in focusing on students' participation in significant out-of-school practices, I failed to consider the practices of mathematicians as my primary point of reference (cf. J. S. Brown et al., 1989). Alternatively, it might be argued that my concern for equity in students' access to significant mathematical ideas is inadequate in that I failed to adopt an social justice agenda (cf. Frankenstein, 2002; Gutstein, in press). In responding to both critiques, I would be obliged to give additional justifications to back up the types of goals I have proposed. The process of specifying and clarify prospective endpoints for students' learning inherent in such exchanges is analogous to the type of reasoning that Kuhn (1962) argued is involved in choices between competing research traditions. In both cases, the type of rationality involved "is a judgmental activity requiring imagination, interpretation, the weighing of alternatives, and application of criteria that are essentially open" (Bernstein, 1983, p. 56). Kuhn contrasted subjectivity with the process of developing supporting reasons or warrants that satisfy communal standards of rationality. Furthermore, he argued that these standards are, as Rorty (1979) put it, "hammered out" by members of an intellectualers of an intellectual community as they pursue their collective enterprise. Viewed in these terms, it is quite possible for mathematics educators to make rational decisions about instructional goals while acknowledging that what counts as improvement in students' mathematical learning is itself open to revision.

The view of that I have proposed of mathematics education as a design science gives rise to a second criterion for comparing and contrasting background theoretical perspectives that concerns their usefulness. Stated as directly as possible, the usefulness criterion focuses on the manner in which different theoretical perspectives might contribute to the collective enterprise of developing, testing, and revising designs for supporting learning. The illustrations I gave of design at the levels of a national educational system, a school or school district, and a classroom indicate that it will be important to consider for whom particular perspectives might do useful work when comparing perspectives with respect to this criterion. For example, a perspective that does useful work for a classroom teacher responsible for supporting the mathematical learning of specific groups of students might not be necessarily be valuable for a school or district administrator whose primary concern is with the learning outcomes of a relatively large student population. This second criterion reflects the view that the choice of theoretical perspective requires pragmatic justification. In a later section of this chapter, I sharpen this criterion by drawing on Dewey's account of pragmatic justification. First, however, I pause to consider traditional epistemological distinctions that have plagued discussions of philosophy in mathematics education. In doing so, I outline an alternative position that Putnam (1987) terms internal or pragmatic realism.

PRAGMATIC REALISM

Discussions of philosophical issues in mathematics education have focused primarily on epistemology in recent years. Epistemology is concerned with inquiries into both the nature of knowledge and the process by which we come to know. It therefore addresses questions that center on the origin, nature, limits, methods, and justification of human knowledge (Hofer, 2002). These discussions were initially sparked in large part by the increasing influence of radical constructivism in mathematics education research in the 1980s. The general *psychological* contention that learning is an active, constructive process is common to numerous variants of constructivism and has become widely accepted. A key distinguishing feature of radical constructivism is the manner in which it supplements this psychological contention with the *epistemological* assertion that it is impossible to check whether our ideas and concepts correspond to external reality (von Glasersfeld, 1984). It is important to emphasize that radical constructivists do not deny the existence of a pre-given external reality. Their central epistemological claim is instead that this reality is unknowable precisely because it is exists independently of human thought and action. Radical constructivists therefore speak of knowledge that has proven viable as fitting with, rather than matching, external reality in that it satisfies the constraints of that reality as much as a key satisfies the constraints of a particular type of lock (von Glasersfeld & Cobb, 1984).

Viewed within a broader philosophical context, radical constructivism is a recent variant of a long line of skeptical epistemologies that can be traced back to the sophists of ancient Greece. Radical constructivism, like other skeptical positions, challenges the traditional project of epistemology, that of identifying a universal method for determining whether a particular theory or conceptual scheme matches or corresponds with external reality. The controversy that radical constructivism has evoked within the mathematics education community stems from the manner in which it, as a skeptical position, plays on the fear that unless we overcome human fallibility and achieve finality in our knowledge claims, we have achieved nothing (Bernstein, 1983; Latour, 2000). The pragmatic realist position that I will outline also seeks to end the traditional epistemological project. However, whereas radical constructivism contends that it is impossible to bridge the gulf between knowledge and reality, pragmatic realism questions a fundamental assumption common to both traditional epistemology and to skeptical positions such as radical constructivism. I can best clarify these two contrasting approaches by focusing on the notion of *reality* that underpins the arguments of both philosophical skeptics and their realist counterparts.

The reality on which realists and skeptics both focus their attention is not populated with tables and chairs, students and teachers, or differential equations and geometry proofs. It is instead an imagined, or perhaps better, an imaginary realm that has been the center of philosophical debate since the time of Plato. Putnam (1987) refers to it as Reality with a capital "R" to distinguish it from the world in which our lives take on significance and meaning. As Bernstein (1983) observes, although skeptics seek an end to the traditional epistemological project, they are also more than willing to play the traditional epistemological game. For all their differences, realists and skeptics agree on the basic image of people as knowers separated from Reality (with a capital "R"). Radical constructivists, for example, question neither the relevance of this notion of an imaginary, ahistorical Reality nor the value of debating whether it is knowable. Putnam (1987), in contrast, follows John Dewey in challenging the dichotomy between a putative external Reality on the one hand, and the concepts and ideas that people use to think about and discuss it on the other. In contrast to radical constructivism, his goal is not to offer new solutions to traditional epistemological problems, but to question the problems themselves. In his view, realists and skeptics are both enthralled by what Dewey (1910/1976) termed "the alleged discipline of epistemology." The appropriate response to both realists and skeptics is therefore not to join them in their debate, but to challenge their exchange as an academic exercise of limited relevance.

For Putnam and Dewey, and indeed for a number of influential twentieth century philosophers including Goodman (1978), Quine (1992), and Taylor (1995), the world is not a screen shutting off nature but a path into it. As Dewey (1929/1958) put it, "experience is *of* as well as *in* nature. It is not experience that is experienced, but nature" (p. 12, italics in the original). In speaking of people being in nature, Dewey displaced the traditional preoccupation with Reality with a focus on people's activities in the realities in which they actually live their lives. As Putnam clarifies, the quest for certainty is given up once this focus is adopted in favor of understanding how people are able to produce fallible truths and achieve relative security in their knowledge claims.

In Putnam's view, the traditional epistemological goal of prescribing to scientists and non-scientists alike how they ought to reason is unjustifiable self-aggrandizement. Like Dewey, his primary concern is instead with processes of inquiry as they are enacted by flesh-and-blood people. Putnam's contribution to this pragmatic line of thought is particularly relevant to my concerns in this chapter because he addresses the issue of how people cope with the multiplicity of perspectives that characterize various domains and social contexts.

As Putnam observes, contemporary science has taken

> away foundations without providing a replacement. Whether we want to be there or not, science has put us in a position of having to live without foundations. . . . That there are ways of describing what are (in some way) the "same facts" which are (in some way) "equivalent" but also (in some way) "incompatible" is a strikingly non-classical phenomenon. (p. 29)

Putnam goes on to note that scientists in many fields switch flexibly from one perspective to another and treat each set of facts as real when they do so. On this basis, he concludes that these scientists *act as* conceptual relativists who treat what counts as relevant facts and as legitimate ways of describing them as relative to the background theoretical perspective that they adopt (cf. Sfard, 1998). In making this claim, he does not focus on scientists' commentaries on the practices of their chosen disciplines. Instead, he uses the pragmatic criterion employed by Dewey, Charles Sanders Peirce, and particularly by William James, namely that the indicator of what people actually believe is not what they say about their activity (i.e., espoused belief) but the suppositions and assumptions on which they risk acting. In the case at hand, scientists risk acting on the assumption that the differing constellations of phenomena that they investigate when they adopt differing theoretical perspectives are each real (with a small "r"). In taking this stance, Putnam rejects as irrelevant the traditional epistemological project of determining which of these constellations of phenomena correspond to an imaginary Reality. He terms his position *pragmatic realism* in that he takes at face value the realities that scientists investigate. In his view, questions concerning the existence of abstract mathematical and scientific entities should be addressed not by making claims about Reality (with a capital "R") but by examining disciplinary practices. In this regard, he notes approvingly that Quine (1953)

> urges us to accept the existence of abstract entities on the ground that these are indispensable in mathematics, and of microparticles and space-time points on the ground that these are indispensable in physics; and what better justification is there for accepting an ontology than its indispensability in our scientific practice? (Putnam, 1987, p. 21)

Putnam makes it clear that pragmatic realism is not limited to mathematics and science.

> [It] is, at bottom, just the insistence that realism is *not* inconsistent with conceptual relativity. One can be *both* a realist and a conceptual relativist. Realism (with a small "r") … is a view that takes our familiar common sense [or everyday] scheme, as well as our artistic and scientific and other schemes, at face value, without helping itself to the thing 'in itself' [i.e., Reality with a capital "R"]. (p. 17, italics in the original)

He goes on to clarify that the world looks both familiar and different when we reject the dichotomy between Reality and people's ideas about it.

> It looks familiar, insofar as we no longer try to divide up mundane reality into a 'scientific image' and a 'manifest [everyday] image' (or our evolving doctrine into a 'first-class' and a 'second-class' conceptual system). Tables and chairs . . . exist just as much as quarks and gravitational fields. . . . The idea that most of mundane reality is an illusion (an idea that has haunted Western philosophy since Plato . . .) is given up once and for all. But mundane reality looks different, in that we are forced to acknowledge that many of our familiar descriptions reflect our interests and choices. (Putnam, 1987, p. 37)

Putman takes care to differentiate *conceptual relativism*, the notion that the phenomena that are treated as real differ from one perspective to another, from the view that every perspective is as good as every other. "Conceptual relativity sounds like 'relativism', but it has none of the 'there is no truth to be found . . . "true" is just a name for what a bunch of people can agree on' implications of relativism" (1987, pp. 17–18). Putnam instead characterizes truths as fallible, historically contingent, human productions that are subject to correction. His goal in adopting this view is to rehabilitate the notion of truth while simultaneously rejecting the view that Truth should be ascertained in terms of correspondence with Reality. In his view, it is imperative to preserve the notion of truth given that people make and risk acting on the basis of their truth claims in both mundane and scientific realities.

Putnam demonstrates convincingly that scientists and the proverbial person in the street both *act as* conceptual relativists. However, in the cases that he considers, the various perspectives are well established and are not subject to dispute. They therefore contrast sharply with the fluid situation in mathematics education research where theoretical perspectives continue to proliferate and are openly contested. Pragmatic realism is nonetheless relevant in that it provides an initial orientation as we begin to address the meta-issue of how we might compare and contrast different perspectives. For example, it leads us to question claims made by adherents to a particular perspective that their viewpoint gets the world of teaching and learning right—that the phenomena that they take as real correspond to Reality. Arguments of this type involve what Putnam terms the fiction that we can usefully imagine a God's Eye point of view from which we can decide which perspective matches Reality. Pragmatic realism instead orients us to acknowledge that we make choices when we adopt a particular theoretical perspective, and that these choices reflect particular interests and concerns. In addition, pragmatic realism alerts us to the danger of inferring from the conclusion that there is no neutral algorithm of theory choice that any theoretical perspective is as good as any other. In Putnam's view, we can counter this "anything goes" claim by making the criteria that we use when justifying our theoretical choices an explicit focus of scrutiny and discussion. It is to contribute to such a discussion that I have proposed two potentially revisable criteria for comparing and contrasting background theoretical perspectives in mathematical education research. The first criterion is concerned with the nature of the realities that are investigated by researchers who adopt different theoretical perspectives whereas the second focuses on the extent to which research conducted within a particular perspective can contribute to the enterprise of formulating, testing, and revising conjectured designs for supporting envisioned learning processes. In the next two sections of this chapter, I sharpen each of these criteria in turn.

THE NOTION OF THE INDIVIDUAL AS CONCEPTUALLY RELATIVE

Historically, mathematics education researchers looked to cognitive psychology as a primary source of theoretical insight. However, during the last fifteen years, a number of theoretical perspectives that treat individual cognition as socially and culturally situated have become increasingly prominent. In their historical overview of the field, De Corte et al. (1996) speak of first wave and second wave theories to distinguish between these two types of theories. The goal of first-wave theories is to model teachers' and students' individual knowledge and beliefs by positing internal cognitive structures and processes that account for their observed activity. They portray the emergence of second-wave theories as a response to the limited emphasis on affect, context, and culture in first-wave research. Theories of this type typically treat teachers' and students' cognitions as situated with respect to their participation in particular social and cultural practices.

The distinction that De Corte et al. (1996) draw between cognitive and situated perspectives has become institutionalized within the mathematics education research community and serves as the primary way in which we compare and contrast theoretical perspectives. These comparisons focus on the extent to which particular perspectives take individual cognitive processes or collective social and cultural processes as primary. Although these comparisons throw important differences between perspectives into sharp relief, they are misleading in one important respect. As I will illustrate when I compare the four background theories of experimental psychology, cognitive psychology, sociocultural theory, and distributed cognition, the notion of the individual is conceptually relative. Adherents to these different perspectives conceptualize the individual in fundamentally different ways. Furthermore, these differences are central to the types of questions that adherents to the four perspectives ask, the nature of the phenomena that they investigate, and the forms of knowledge they produce. I will therefore concretize the first of the two criteria that I have proposed for comparing and contrasting theoretical perspectives by teasing out these differences.

I can best exemplify the difficulties that arise when the notion of the individual is taken as self evident by previewing my discussion of distributed theories of intelligence. The term distributed intelligence is perhaps most closely associated with Pea (1985; 1987; 1993). A central assumption of this theoretical perspective is that intelligence is distributed "across minds, persons, and symbolic and physical environments, both natural and artificial" (Pea, 1993, p. 47). Dörfler (1993) clarified the relevance of this theoretical perspective for design and research in mathematics education when he argued that thinking

> is no longer considered to be located exclusively within the human subject. The whole system made up of the subject and the available cognitive tools and

> aids realizes the thinking process. . . . Mathematical thinking for instance not only *uses* those cognitive tools as a separate means but they form a constitutive and systematic part of the thinking process. The cognitive models and symbol systems, the sign systems, are not merely means for expressing a qualitatively distinct and purely mental thinking process. The latter realizes itself and consists in the usage and development of the various cognitive technologies. (p. 164)

This theoretical orientation is consistent with the basic Vygotskian insight that students' use of symbols and other tools profoundly influences both the process of their mathematical development and its products, increasingly sophisticated mathematical ways of knowing (Dörfler, 2000; Hall & Rubin, 1998; Kaput, 1994; Lehrer & Schauble, 2000; Meira, 1998; van Oers, 2000).

In developing his distributed perspective, Pea (1993) has been outspoken in delegitimizing analyses that take the individual as a unit of analysis. In his view, the functional system consisting of the individual, tools, and social contexts is the appropriate unit. Not surprisingly, Pea's admonition has been controversial. For example, Solomon (1993) responded by arguing that, in distributed accounts of intelligence, "the individual has been dismissed from theoretical consideration, possibly as an antithesis to the excessive emphasis on the individual by traditional psychology and educational approaches. But as a result the theory is truncated and conceptually unsatisfactory" (p. 111). It is worth pausing before adopting one or the other of these opposing positions to clarify what Pea might mean when he speaks of the individual. His arguments indicate that individuals, as he conceptualizes them, do not use tools and do not take account of context as they act and interact. They appear to be very much like individuals as portrayed by mainstream cognitive science who produce observed behaviors by creating and manipulating internal symbolic representations of the external environment. Pea's proposal is to equip these encoders and processors of information with cultural tools and place them in social context. This proposition is less contentious than his apparent claim that all approaches that focus on the quality of individuals' reasoning should be rejected regardless of how the individual is conceptualized. The relevant issue once we clarify what we are talking about is not whether it is legitimate to focus on individual teachers' and students' reasoning. Instead, it is how we might usefully conceptualize the individual given our concerns and interests as mathematics educators.

USEFULNESS AND TRUTH

A concern for the usefulness of different theoretical perspectives carries with it the implication that they can be viewed as conceptual tools. This metaphor fits well with the characterization of mathematics education as a design science and orients us away from making forced choices between one perspective and another. However, for this criterion to itself do useful work, we have to clarify what it means for a theoretical perspective to contribute to the collective enterprise of mathematics education. Prawat's (1995) discussion of three types of pragmatic justification identified by Pepper (1942) is relevant in this regard.

The first type of pragmatic justification is purely instrumental in that actions are judged to be true if they enable the achievement of goals. Prawat (1995) indicates the limitations of this formulation when he observes that "a rat navigating a maze has as much claim on truth, according to this approach, as the most clear-headed scientist" (p. 19). In the second type of pragmatic justification,

> it is not a successful act that is true, but the hypothesis that leads to the successful act. When one entertains a hypothesis, Pepper (1942) points out, it is in anticipation of a specific outcome. When that outcome occurs, the hypothesis is verified or judged true. (Prawat, 1995, p. 19)

As Prawat goes on to note, this is a minimalist view in that hypotheses are treated as tools for the control of nature. This portrayal sits uncomfortably with Tolumin's (1963) demonstration that the primary goal of science is to develop insight and understanding into the phenomena under investigation, and that instrumental control is a by-product.

The third type of pragmatic justification, which Pepper (1942) calls the qualitative confirmation test of truth, brings the development of insight and understanding to the fore. This type of justification builds on Dewey's analysis of the function of thought and the process of verification. Dewey viewed ideas as potentially revisable plans for action and argued that their truth is judged in terms of the extent to which they lead to a satisfactory resolution to problematic situations (Westbrook, 1991). In taking this stance, he equated experience with physical and conceptual action in a socially and culturally organized reality, and maintained that it is characterized by a future-oriented projection that involves an attempt to change the given situation (Sleeper, 1986; J. E. Smith, 1978). He therefore contended that foresight and understanding are integral to thought, the primary function of which is to project future possibilities and to prepare us to come to grips with novel, unanticipated occurrences.

These aspects of thought are central to Dewey's analysis of verification as a process in which the phenomena under investigation talk back, giving rise to surprises and inconsistencies. He clarified that some of these surprises can be accounted for relatively easily by elaborating underlying ideas, whereas others constitute conceptual impasses that typically precipitate either the reworking of ideas or their eventual rejection (Prawat, 1995). The crucial point to note is that, for Dewey, verification constitutes a context in which theoretical ideas are elaborated and modified. As Pepper (1942) put it, ideas judged to be true are those that give *insight* into what he termed the texture and quality of the phenomena that serve to verify them. In the case of mathematics education, for example, ideas are potentially useful to the extent that they give rise to conjectures about envisioned learning processes and the specific means of supporting them. However, these ideas are not simply either confirmed unchanged or rejected during the process of testing and revising designs. Instead, Dewey drew attention to the process by which ideas that are confirmed evolve as they are verified. In his formulation, *the truth of fallible, potentially revisable ideas is justified primarily in terms of the insight and understanding they give into learning processes and the means of supporting their realization.* This is the criterion that I will use when I focus on the potentially useful work that experimental psychology, cognitive psychology, sociocultural theory, and distributed cognition might do in contributing to the enterprise of formulating, testing, and revising conjectured designs for supporting envisioned learning processes.

As a point of clarification, it is important to note that Dewey's intent in analyzing verification processes was descriptive rather than prescriptive. He was not attempting to specify rules or norms for verification to which scientists ought to adhere. Instead, he sought to understand verification as it is actually enacted in the course of inquiry. He claimed that regardless of the pronouncements of philosophers who strive to identify the Method for determining Truth, verification as it is enacted by scientists is an interpretive process in which they modify and elaborate theoretical ideas. Dewey's analysis has far reaching implications in that it challenges a distinction central to the traditional project of epistemology, that between the context of discovery and the context of justification. As Bernstein (1983) explains,

> while no contemporary philosopher of science has wanted to claim that there is a determinate decision procedure or method for advancing scientific discovery, many have been firmly convinced that there are (or ought to be) permanent procedures for testing and evaluating rival theories. This is the basis of the "orthodox" distinction between the context of discovery and the context of justification. The latter has been taken to be the proper domain of the philosopher of science. His or her task is to discover, specify and reconstruct the criteria. (p. 70)

Popper's (1972) contention that to be scientific, hypotheses and theory have to be open to falsification is the most widely cited proposal of this type. As a rough rule of thumb, Dewey would probably not have disputed Popper's claim. However, his analysis of verification indicates that he would add that the application of this rule necessarily involves wisdom and judgment. In this regard, he would concur with Bernstein's observation that

Popper frequently writes as if we always know in advance what will count as a good argument or criticism against a conjecture. The basic idea behind the appeal to falsification as a demarcation criterion between science and nonscience is that there are clear criteria for determining under what conditions a conjecture or hypothesis is to be rejected. (p. 70)

As Dewey pointed out and as Kuhn (1962; 1977), Lakatos (1970), and Feyerabend (1975) subsequently underscored, the situation becomes less straightforward than Popper's proposal implies when we focus on the actual practices of scientists. Although the phenomena under investigation talk back, they do not serve as a jury that tell us unambiguously whether a theoretical idea should be accepted or rejected.

We frequently do not know, in a concrete scientific situation, whether we are confronted with an obstacle to be overcome, a counterinstance that can be tolerated because of the enormous success of the theory, or with evidence that should be taken as falsifying our claim. Data or evidence do not come marked "falsification"; in part, it is we who decide what is to count as a falsification or refutation. (Bernstein, 1983, p. 71)

Furthermore, as Dewey in particular emphasized, the ideas under scrutiny are moving targets that evolve as scientists, in effect, engage in a dialogue with the phenomena under investigation.

Latour and Wollgar's (1979) analysis of activity in an organic chemistry laboratory and Pickering's (1984) of a high energy physics laboratory substantiate Dewey's analysis of verification, particularly as the work carried out in both laboratories resulted in Nobel prizes. The case studies reveal that the scientists did not simply formulate conjectures, conduct experiments to test them, and then passively let the resulting data determine which were confirmed and which were false. To be sure, the scientists had theoretical ideas and conjectures expressed in terms of those ideas. However, they also had views about how the experimental apparatus functioned, how it could be used, and so forth. As Latour and Wollgar, and Pickering both document, it was typical for the apparatus to initially not behave as the scientists expected. In these situations, the scientists attempted to adjust to the perceived anomaly in a number of ways that included revising the theory under investigation, revising their views of how the apparatus functioned, or tinkering with and rebuilding the apparatus itself (Traweek, 1988). Their goal in doing so was to achieve what Pickering (1984) terms a robust fit between these different aspects of the experimental situation. Furthermore, the scientists judged the theoretical ideas that they successfully revised and elaborated in the course of this problem-solving process to be true because they gave them insight into the phenomena they were investigating. Latour and Wollgar's and Pickering's analyses of the process by which scientists produce truths as they engage in a dialogue with nature are consistent with Dewey's contention that truths are made through the process by which they are verified (Westbrook, 1991). This pragmatic notion of truth and, relatedly, of usefulness underpins my comparison of different theoretical perspectives in terms of their potential to contribute to the enterprise of formulating, testing, and revising conjectured designs for supporting envisioned learning processes.

COMPARING THEORETICAL PERSPECTIVES

My purpose in comparing and contrasting the four theoretical perspectives is to illustrate the relevance of the two criteria I have proposed: (a) How the individual is conceptualized in the differing perspectives, and (b) the potential of the perspectives to contribute to our understanding of learning processes and the means of supporting their realization. I therefore make no pretense at providing a comprehensive overview of theoretical perspectives in mathematics education. For example, I largely ignore both symbolic interactionism (Bauersfeld, 1980, 1988; P. Cobb, Wood, Yackel, & McNeal, 1992; Voigt, 1985, 1996) and discourse and communicational perspectives (Ernest, 1994; Pimm, 1987, 1995; Rotman, 1988, 1994; Sfard, 2000a, 2000b). Experimental psychology refers to the psychological research tradition in which the primary methods employed involve experimental and quasi-experimental designs, preferably with the random assignment of subjects. My decision to focus on this perspective has been influenced by the strong advocacy for research designs of this type by personnel in the Institute of the Educational Sciences and other U.S. government funding agencies.

My discussion of cognitive psychology is limited to theoretical orientations that involve what MacKay (1969) termed the actor's viewpoint. The goal of psychologies of this type is to account not merely for teachers' and students' observed behaviors but for their inferred interpretations and understandings in terms of internal cognitive structures and processes. This restricted focus is premised on the observation that the relationships we establish with teachers and students in the course of our work as mathematics educators frequently involve collaboration and mutual engagement. As Rommetveit (1992) and Schutz (1962) both illustrate, relationships of this type involve communicative interactions characterized by a reciprocity of perspectives typical of the actor's viewpoint. I will therefore have little to say about cognitive approaches that involve the observer's viewpoint (MacKay, 1969) and that focus on internal cognitive processes that intervene between an observed stimulus environment and observed response activity. Sociocultural theory and distributed cognition are both second wave perspectives as described by De Corte et al. (1996) and provide points of contrast with cognitive psychology by viewing individual activity as situated with respect to social and cultural practices. As I will clarify, sociocultural theory has developed largely independently of western cognitive psychology by drawing inspiration directly from the writings of Vygotsky and Leont'ev whereas distributed cognition has emerged in reaction to information-processing psychology and incorporates aspects of the Soviet work.

Experimental Psychology

In speaking of experimental psychology, I refer to the psychological research tradition whose primary contributions to mathematics education have involved the development of assessment instruments, particularly norm-referenced tests, and the findings of studies that have assessed the relative effectiveness of alternative curricular and instructional approaches. This perspective merits attention given the far reaching implication of a recent legislative initiative in the United States known as the No Child Left Behind (U.S. Congress, 2001). As Slavin (2002) observes, the Act "mentions 'scientifically based research' 110 times. It defines 'scientifically based research' as 'rigorous, systematic and objective procedures to obtain valid knowledge,' which includes research that 'is evaluated using experimental or quasi-experimental designs,' preferably with random assignment" (p. 15). Slavin goes on to note that the recently established Institute for the Educational Sciences, a major federal funding agency, is organized "to focus resources on randomized and rigorously matched experimental research on programs and policies that are central to the education of large numbers of children" (p. 15). Whitehurst (2003, April), the Institute's Director, addresses this point when he states that although "interpretations of the results of randomized trials can be enhanced with results from other methods" (p. 9), "randomized trials are the only sure method for determining the effectiveness of education programs and practices" (p. 6). In a nutshell, "randomized trials are the gold standard for determining what works" (p. 8). As Whitehurst's reference to "what works" indicates, the emphasis on a particular research method profoundly influences both the nature of the questions asked and the forms of knowledge produced. In this regard, Slavin (2004) clarifies that well-designed studies of this type are not limited to *x* versus *y* comparisons but can "also characterize the conditions under which *x* works better or worse than *y*, the identity of the students for whom *x* works better or worse than *y*, and often produce rich qualitative information to supplement the quantitative comparisons" (p. 27).

Knowledge claims of this type are premised on a particular conception of the individual. In teasing out this conception and thus the nature of the reality that is the focus of investigation, it is important to distinguish between the abstract, collective individual to whom knowledge claims refer and the individual students who participate in experiments. This collective individual is a statistical aggregate that is constructed by combining measures of psychological attributes of the participating students (e.g., measures of mathematical competence as measured by achievement test scores). As Danziger (1990) demonstrates in his seminal historical analysis of experimental psychology, the purpose of experimentation is to make predictions about how certain variations in instructional conditions affect the performance of this abstract individual as assessed by aggregating the scores of individual students. This statistically constructed individual is abstract in the sense that it need not correspond to any particular student. The construction of this collective individual enables investigators who work in this tradition to avoid the issue of individual differences and the challenges involved in accounting for the reasoning and learning of specific students (Danziger, 1990). This is accomplished by treating differences in measures of students' performance as error variance. Students are then characterized by the extent to which their performance deviates from group norms that measure the performance of the collective individual.

This methodological approach of investigating the performance of the abstract, collective subject rests on two underlying assumptions. The first is that students are composed of discrete, isolatable attributes or qualities that vary only in degree from one student to another (Danziger, 1990). This assumption makes it legitimate to combine measures of students' performance. Students so portrayed are therefore limited to possessing larger or smaller measurable amounts of these psychological attributes. Second, the environments in which students acquire these capacities are composed of independent features that the investigator can manipulate and control directly. The implicit ontology is that of environmental settings made up of separate independent variables and students composed of collections of dependent psychological attributes. Together, these two theoretical suppositions ground investigations that seek to discern causal relationships between the manipulation of instructional conditions on the performance of the collective, abstract individual. This theoretical underpinning indicates that experimental psychology should be treated as a theoretical perspective rather than merely as a set of methodological prescriptions.

The forcefulness with which adherents of this perspective have attempted to advance their viewpoint has elicited a number of responses. One line of critique claims that the approach of casting students as objects in studies that focus on their responses to environmental manipulations is ethically dubious and potentially dehumanizing. As these critiques target the morality of investigators working in this tradition, it is worth noting with Porter (1996) that

> social quantification means studying people in classes, abstracting away their individuality. This is not unambiguously evil, though of late it has been much criticized. Much, probably most, statistical study of human populations has aimed to improve the condition of working people, children, beggars, criminals, women, or racial and ethnic minorities. (p. 77)

A second line of attack develops the argument that experimental psychology lacks theoretical depth. For example, Danziger (1990) contends that this analytic approach both separates people from the social contexts in which their actions take on significance, and eschews the study of the interpretations that they make in those contexts. Criticisms of this type miss the mark in my view because they use the theoretical commitments of alternative theoretical perspectives as criteria against which to assess experimental psychology. Judgments of theoretical depth do not transcend research traditions but are instead conceptually relative to the norms and values of particular research communities. Experimental psychologists, for example, make judgments of theoretical depth by gauging the extent to which the findings of investigations contribute to their collective understanding of the psychology of the abstract, statistically constructed individual. More generally, exchanges between experimental psychologists and their opponents typically involve people talking past each other as they point to features of the different realities that they investigate. In my view, it is more productive to assess the value of experimental psychology in terms of its potential contributions to a specific enterprise such as that of formulating, testing, and revising conjectured designs for supporting mathematical learning. There is, however, one issue that requires additional scrutiny before we consider the potential usefulness of this theoretical perspective.

Advocates of experimental psychology frequently argue with considerable vehemence that their perspective and its associated research methods are scientific, and that all other perspectives and methodological approaches are not. As the case studies conducted by Pickering (1984) and by Latour and Woolgar (1979) indicate, this claim is not based on an examination of the practices of scientists whose work is widely acknowledged to be of the highest caliber. The claim instead capitalizes on folk beliefs about what it means to be scientific and objective. In the history of the discipline, what was important

> was the widespread acceptance of a set of firm convictions about the nature of science. To be socially effective, it was not necessary that these convictions actually reflected the essence of successful scientific practice. In fact, most popular beliefs in this area were based on external and unanalyzed features of certain practices in the most prestigious parts of science. Such beliefs belong to the rhetoric of science rather than its substance. . . . Such unquestioned emblems of scientific status included features like quantification, experimentation, and the search for universal (i.e., ahistorical) truths. (Danziger, 1990, p. 120)

It is instructive to compare the features of scientific practice that advocates of experimental psychology deem to be critical with those identified by the National Research Council (2002):

> *Scientific Principle 1*
> Pose Significance Questions That Can Be Investigated Empirically
>
> *Scientific Principle 2*
> Link Research to Relevant Theory
>
> *Scientific Principle 3*
> Use Methods That Permit Direct Investigation of the Question
>
> *Scientific Principle 4*
> Provide a Coherent and Explicit Chain of Reasoning
>
> *Scientific Principle 5*
> Replicate and Generalize Across Studies
>
> *Scientific Principle 6*
> Disclose Research to Encourage Professional Scrutiny and Critique (pp. 3–5)

Eisenhart and Towne (2003) clarify that these principles were identified by “reviewing *actual* research programs—both basic and applied—in natural science, social science, education, medicine, and agriculture” (p. 33, italics in the original). As they note, scientifically based research is best defined not by the employment of particular research methods but by characteristics that cut across a range of methods. I would only add that the use of these or any other set of principles to assess specific research programs necessarily involves interpretations and judgments of type that all scientists including experimental psychologists make in the course of their practice.

It is ironic that the claim that experimental psychology has hegemony over what is regarded as scientific lacks empirical grounding and is contradicted by the available evidence. This claim is ideological in the pernicious sense of the term. It rests on the indefensible proposition that the methods of experimental psychology are theory neutral and constitute the only guaranteed means of gaining access to Reality with a capital “R”. As I have demonstrated, experimental psychologists’ use of these methods is theory laden and reflects theoretical suppositions and assumptions about the nature of the reality that they are investigating. In using these methods, they attempt to gain insights into the psychology of the statistically constructed collective individual, a character that is composed of a set of isolatable psychological characteristics and that inhabits a world made up of manipulable independent variables. This observation does not threaten the credibility of experimental psychology as a viable research tradition: The research practices that any research community establishes as normative necessarily entails theoretical commitments. The observations, however, do undermine the claim that experimental psychologists have found the Method for discerning Truth. The failure of many of their most forceful advocates to acknowledge that research in this tradition is conducted from a theoretical perspective closes down debate and, in my view, highlights the political nature of boundary disputes between science and non-science. Adherents to experimental psychology frequently state that their primary motivation in conducting investigations is to improve the students' intellectual and moral welfare. I accept these statements at face value but question the implication that insights developed in any other perspective can make at best marginal contributions to students' intellectual and moral welfare. This brings us to the issue of usefulness.

Adherents of experimental psychology repeatedly emphasize their commitment to conduct research that is pragmatically relevant. For example, Whitehurst (2003, April) stresses that "the primary focus for the Institute [of Educational Sciences] will be on work that has high consideration of use, that is practical, that is applied, that is relevant to practitioners and policy makers" (p. 4). In addressing this and similar claims, it is important to consider for whom the forms of knowledge produced by experimental psychology might be useful. Danziger's (1990) historical analysis is pertinent in this regard. He reports that

> after the turn of the [twentieth] century, psychologists' relations with teachers became increasingly overshadowed by a new professional alliance which was consummated through the medium of a new set of investigative practices. The group of educators with whom psychologists now began to establish an important and beneficial professional alliance consisted of a new generation of professional educational administrators. This group took control of a process of educational rationalization that adapted education to the changed social order of corporate industrialism. The interests of the new breed of educational administrator had little in common with those of the classroom teacher. Not only were the administrators not directly concerned with the process of classroom teaching, they were actually determined to separate their professional concerns as much as possible from those of the lowly army of frontline teachers. In this context, they emphasized scientific research as a basis for the rationalized educational system of which they were the chief architects. In the United States the needs of educational administration provided the first significant external market for the products of psychological research in the years immediately preceding World War I. (p. 103)

Danziger goes on to clarify that knowledge about the responses of the statistically constructed collective individual to different instructional conditions served the needs of administrators who managed institutions in which instruction is carried out in groups. Furthermore, the conception of learning environments as composed of manipulable independent variables was directly relevant to administrators who sought to both distance themselves from and manage classroom instructional processes. Danziger also demonstrates that the consequences of this alliance were at least as profound for experimental psychology as they were for education. For example, it was during this period that the goal of gaining insight into individual mental processes was displaced by that of putting psychological prediction at the service of administrative needs. In addition, the high value that experimental psychologists place on quantification was influenced by the usefulness of statistical constructions based on group data to administrators. On the basis of these observations, Danziger concludes that experimental psychology was transformed in large measure into what he terms an administrative science in that it investigates the reality that administrators seek to manage.

In terms of the metaphor of a theoretical perspective as a conceptual tool, Danziger's analysis indicates that experimental psychology is a tool that has been fashioned to produce forms of knowledge that fulfill administrative concerns. The resulting forms of knowledge enable administrators who are removed from the classroom and who do have little if any specialized knowledge of teaching and learning in particular content domains to make informed decisions about the curricula and instructional strategies that teachers should use. Although these forms of knowledge can be of some relevance to teachers, they are less well suited to the contingencies of classroom teaching. This is because the knowledge produced of individual students is, as Danziger (1990, p. 165) puts it, "a knowledge of strangers" who are known only through their standing in the group. It does not therefore touch on the challenges, dilemmas, and uncertainties that arise in the classroom as teachers attempt to achieve a mathematical agenda while simultaneously taking account of their students' proficiencies, interests, and needs (Ball, 1993; Davis, 1997; Fennema, Franke, & Carpenter, 1993; Lampert, 1990, 2001). I therefore question the frequent assertion made by proponents of experimental psychology that the forms of knowledge it produces are of equal relevance to teachers and administrators. It is significant that the examples that proponents cite to illustrate the need for these forms of knowledge almost invariably concern administrators and policymakers but not classroom teachers. For example, Whitehurst (2003, p. 5) does not mention classroom teachers when he explains that personnel at the Institute of Educational Sciences "recently completed a survey of a purposive sample of our customers to determine what they think we ought to be doing to serve their needs. The sample included school superintendents and principals, chief state school officers, and legislative policy makers" (p. 5).

In summary, the arguments that adherents of experimental psychology advance in support of their perspective are based on the claims that it has hegemony over the production of scientific knowledge about teaching and learning, and that these forms of knowledge are pragmatically useful. I have suggested that the first of these claims is primarily ideological but that the second has merit provided we clarify for whom the resulting forms of knowledge are useful. Proponents of this perspective offer us a bold vision in which studies involving experimental and quasi-experimental designs constitute the primary basis for an ongoing process of educational improvement. It is worth noting that this vision is based on belief rather than evidence and is not supported by the historical record. There was an initial period of enthusiasm for such a vision in the early part of the last century, but this enthusiasm has since waned.

> In spite of these promising beginnings, the history of treatment-group methodology in research in education and educational psychology was not exactly a march of triumph. . . . In due course . . . a certain pessimism about the prospects of experimental research in education began to set in. The claims made on behalf of the quantitative and experimental method had undoubtedly been wildly unrealistic, and in the light of changing priorities the illusions of the early years were unable to survive. (Danziger, 1990, p. 115)

In my judgment, the claim that sustained improvements in mathematics teaching and learning can be made by relying *almost exclusively* on the findings of treatment-group studies is untenable. To be sure well-designed studies of this type can, in all probability, make an important contribution. However, advocates of experimental psychology should, in my view, moderate their rhetoric lest the history of high hopes and dashed expectations repeat itself.

Cognitive Psychology

In comparing and contrasting cognitive psychology with other theoretical perspectives, I restrict my focus to theoretical approaches that seek to account for teachers' and students' inferred interpretations and understandings in terms of internal cognitive structures and processes. Cognitive approaches of this type take on a challenge sidestepped by experimental psychology, that of accounting for specific students' and teachers' mathematical reasoning and learning. It is useful to distinguish between two general types of theories developed within this tradition. The first are theories of the *process* of mathematical learning that are intended to offer insights into students' learning in any mathematical domain, whereas the second are theories of the development of students' reasoning in specific mathematical domains. Pirie and Kieren's (1994) recursive theory of mathematical understanding serves to illustrate theories of the first type. They differentiate a sequence of levels of mathematical reasoning and model mathematical understanding as a recursive phenomenon that occurs as thinking moves between levels of sophistication. In their theoretical scheme, students who are novices to a particular mathematical domain initially make images of either their situation-specific activity or its results. The first significant development occurs when students can take such images as givens and do not have to create them anew each time. Later developments involve students noticing properties of their mathematical images and subsequently taking these properties as givens that can be formalized. One of the notable features of Pirie and Kieren's theory of the growth of mathematical understanding is its broad scope in tracing development from the creation of images to formalization and axiomatization.

Additional examples of theories of the process of mathematical learning include Dubinsky's (1991) theory of encapsulation, Sfard's (1991; Sfard & Linchevski, 1994) theory of reification, Dörfler's (1989) analysis of protocols of action, and Vergnaud's (1982) analysis of the process by which students gradually explicate their initial theorems-in-action. Each of these theorists consider the primary source of increasingly sophisticated forms of mathematical reasoning to be students' activity of interpreting and attempting to complete instructional activities, not the instructional activities themselves. Furthermore, they each characterize mathematical learning as a process in which operational or process conceptions evolve into what Sfard (1991) terms object-like structural conceptions. Proficiency in a particular mathematical domain is therefore seen to involve the conceptual manipulation of mathematical objects whose reality is taken for granted. Greeno (1991) captured this aspect of mathematical proficiency when he introduced the metaphor of acting in a mathematical environment in which tools and resources are ready at hand to characterize number sense. For her part, Sfard (2000b) describes mathematical discourse as a virtual reality discourse to highlight the parallels between this discourse and the ways in which we talk about physical reality. The intent of each of the theoretical schemes I have referenced is to provide a conceptual framework or toolkit that can be used to develop accounts of the process of specific students' mathematical learning (Thompson & Saldanha, 2000).

In contrast to these general conceptual frameworks, theories of the second type focus on the development of students' reasoning in specific mathematical domains. Examples include analyses of early number reasoning (Carpenter & Moser, 1984; Fuson, 1992; Steffe & Cobb, 1988), multiplicative reasoning (Confrey & Smith, 1995; Streefland, 1991; Thompson, 1994; Vergnaud, 1994), geometric reasoning (Clements & Battista, 1992; van Hiele, 1986), algebraic reasoning (Filloy & Rojano, 1984; Kaput, 1999; Sfard & Linchevski, 1994), and statistical reasoning (Konold & Higgins, in press; Mokros & Russell, 1995; Saldanha & Thompson, 2001). It is important to note that these domain specific frameworks do not focus on the mathematical development of any particular student but are instead, like theories of the first type, concerned with the learning of an idealized student that Thompson and Saldanha (2000) refer to as the *epistemic individual*. Researchers working in this cognitive tradition account for variations in specific students' reasoning by using the constructs that comprise their framework to develop explanatory accounts of each student's mathematical activity. This approach enables the researchers to both compare and contrast the quality of specific students' reasoning and to consider the possibilities for their mathematical development.

The common element that ties both types of theories together is their portrayal of both the epistemic individual and of specific students as active constructors of increasingly sophisticated forms of mathematical reasoning. The metaphor of learning as a process of construction can be traced to the eighteenth century Italian philosopher Giambattista Vico who was the first to advance an explicitly constructivist position when he argued that "the known is the made" (Berlin, 1976). Vico's arguments anticipated several of the major claims that the German philosopher Immanuel Kant made 80 years later. In his treatise *A Critique of Pure Reason* (Kant, 1998), Kant contended that our perceptions are always in the form of objects because our minds have a priori structures or intuitions of space and time. He also argued that in addition to being perceived in space and time, objects are experienced through four a priori categories of understanding: quantity (e.g., plurality, totality), quality (e.g., negation), relation (e.g., causality and dependence), and modality (e.g., possibility and necessity).

Kant's philosophical analysis provided the backdrop against which the most significant contributor to constructivism in cognitive psychology, Jean Piaget, conducted his research (Fabricius, 1979). Although Piaget is typically viewed as a psychologist, he described himself as a genetic epistemologist (Piaget, 1970). In this context, the term genetic denotes genesis and encompasses both the origins and development of forms of knowledge. Piaget explained that, ideally, he would have studied the origins and subsequent evolution of foundational categories of understanding as they occurred in the history of humankind. However, as this was impossible, he attempted to gain insight into epistemological issues by studying the development of these notions in children. Significantly, the concepts that he focused on in his investigations correspond almost exactly to the fundamental categories of thought proposed by Kant 150 years earlier. His intent was to account for the development of these categories by relying on constructs such as assimilation, accommodation, and equilibration thereby showing that there is no need to posit that they are *a priori*.

The aspect of Piaget's work that has gained most attention among both psychologists and educators is his claim to have identified a sequence of invariant stages through which children's thinking progresses. The cognitive theorists I have referenced have, in contrast, focused primarily on the process aspects of Piaget's theory. Piaget drew on his early training as a biologist to characterize intellectual development as an adaptive process in the course of which children reorganize their sensory-motor and conceptual activity (Piaget, 1980). In appropriating Piaget's general constructivist orientation to the process of development, cognitive theorists have necessarily had to adapt the theoretical constructs that he proposed given that their concern is to gain insight into the process of students' mathematical learning rather than with problems of genetic epistemology. The resulting characterizations that they propose of the individual differ significantly from that of experimental psychology.

As we have seen, experimental psychology focuses on how variations in instructional conditions affect the performance of the statistically constructed collective subject. In contrast, cognitive psychology focuses on how the epistemic individual successively reorganizes its activity and comes to act in a mathematical environment. Thus, whereas experimental psychologists direct their attention to an environment composed of manipulable independent variables, cognitive psychologists seek to delineate how the world of meaning and significance in which the epistemic individual acts changes in the course of development. We have also seen that experimental psychology characterizes specific students in terms of the deviation of their performance from that of the collective subject. In contrast, cognitive psychology characterizes specific students in terms of the nature or quality of their mathematical reasoning. Thus, whereas experimental psychology is premised on the assumption that students possess large or small amounts of psychological attributes, cognitive psychology is premised on the assumption that students' development involves *qualitative* changes in their mathematical reasoning.

In cases such as this and other cases where characterizations of the individual contrast sharply, it is tempting to try and determine which perspective gets people right. Following Putnam (1987), I have argued that this quest is misguided and that it is more productive to compare and contrast differing theoretical perspectives in terms of their potential usefulness. In this regard, I have suggested that the forms of knowledge produced by experimental psychology are particularly useful to administrators who are responsible for managing educational systems at some distance removed from the classroom and who have little grounding in classroom teaching and learning processes in particular content domains. Given their administrative concerns and interests, it is unlikely that they would see value in the forms of knowledge produced by cognitive psychologists working in the tradition on which I have focused. Conversely, having conducted studies that involve experimental designs (P. Cobb et al., 1991; P. Cobb, Wood, Yackel, & Perlwitz, 1992), I can attest that the resulting forms of knowledge are not well suited to the demands of instructional design at the classroom level. A fundamental difficulty is that studies involving experimental designs do not produce the detailed kinds of data that are needed to guide the often subtle refinements made when improving an instructional design at this level.

Proponents of the cognitive tradition that I have discussed frequently assume that the types of explanatory frameworks they produce constitute an adequate basis for both classroom instructional design and pedagogical decision making. I have contributed to the development of such a framework (Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983) and have attempted to use it to guide the development and refinement of classroom instructional designs. On the basis of this experience, I question the claim that such frameworks are, by themselves, sufficient. The primary difficulty is precisely that the forms of knowledge produced are cognitive rather than instructional and do not involve positive heuristics for design (Gravemeijer, 1994b). In my view, researchers working in this tradition have inherited perspectives and associated methodologies from cognitive psychology but have not always reflected on the relevance of the forms of knowledge produced to the collective enterprise of mathematics education. The issue of how these perspective and methodologies might be adapted so that they can better contribute to the concerns and interests of mathematics education has rarely arisen because their sufficiency has been assumed almost as an article of faith (see Thompson, 2002, for a rare discussion of these issues).

Given that cognitive theories of mathematical learning do not, by themselves, constitute a sufficient basis for design, the question of clarifying the contributions that they can make remains. Drawing on my experience of developing and revising designs at the classroom level, I can identify three contributions of theories of development in particular mathematical domains that I do not claim are exhaustive. First, domain-specific theories typically include analyses of the forms of reasoning that we want students to develop. These analyses are non-trivial accomplishments and can serve to specify the "big mathematical ideas," thereby giving an overall orientation to the instructional design effort. Second, domain-specific frameworks can alert the designer to major shifts in students' mathematical reasoning that the design should support. Third, the designer's or teacher's use of a domain-specific framework to gain insight into specific students' mathematical reasoning can inform the design of instructional activities intended to support subsequent learning. This use of a cognitive framework is particularly evident in the successful Cognitively Guided Instruction program developed by Carpenter, Fennema, and colleagues (Carpenter & Fennema, 1992; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1993). As I will clarify when I discuss distributed cognition, I consider this emphasis on instructional activities as the primary means of supporting students' mathematical learning to be overly restrictive. It is also worth noting that domain-specific cognitive frameworks do not orient designers to consider issues of equity in students' access to significant mathematical ideas even though the resulting designs can, on occasion, make a significant contribution in this regard (Carey, Fennema, Carpenter, & Franke, 1995; Silver, Smith, & Nelson, 1995).

In summary, general and domain-specific theories of mathematical learning are both concerned with the learning of an idealized student, the epistemic individual. Variations in specific students' reasoning are accounted for by using the constructs central to theories of this type to develop explanations of their mathematical activity. Although there are substantive differences between the various cognitive theories I have referenced, they are tied together by the characterization of mathematical learning as a constructive process in the course of which students successively reorganize their sensory-motor and conceptual activity. From the perspective of experimental psychology, this multiplicity of theories is frequently taken as evidence that the cognitive tradition on which I have focused is not scientific. It is therefore worth reiterating that experimental psychologists have achieved unanimity only by eschewing a concern for cognitive processes and structures that account for observable performance and by treating variation in performance as error variance. Whereas the forms of knowledge produced by experimental psychology are well suited to the concerns of administrators, cognitive theories can contribute to the development and improvement of classroom instructional designs.

Sociocultural Theory

At their core, the forms of knowledge produced by the cognitive theorists on whose work I have focused concern the process by which the epistemic individual successively reorganizes its activity. In contrast, the forms of knowledge produced by sociocultural theorists concern the process by which people develop particular forms of reasoning as they participate in established cultural practices. This theoretical perspective treats intellectual development and the process by which people become increasingly substantial participants in various cultural practices as aspects of a single process. Consequently, whereas cognitive theorists investigate the activity of the epistemic individual, sociocultural theorists investigate the participation of the *individual-in-cultural-practice*. In a very real sense, the two groups of researchers are attempting to understand different realities. It is necessary to clarify the origins of sociocultural theory in order to understand the reality into which it gives insight.

Contemporary sociocultural theory draws directly on the writings of Vygotsky and Leont'ev. Vygotsky (1962; 1978; 1981) made his foundational contributions to this perspective during the period of intellectual ferment and social change that followed the Russian revolution. In doing so, he was profoundly influenced by Marx's argument that it is the making and use of tools that serves to differentiate humans from other animal species. For Vygotsky, human history is the history of artifacts such as language, counting systems, and writing that are not invented anew by each generation but are instead passed on and constitute the intellectual bequest of one generation to the next. In formulating his theory of intellectual development, Vygotsky developed an analogy between the use of physical tools and the use of intellectual tools such as sign systems (Kozulin, 1990; van der Veer & Valsiner, 1991). His central claim was that just as the use of a physical tool serves to reorganize activity by making new goals possible, so the use of sign systems serves to reorganize thought. He viewed culture as a repository of sign systems and other artifacts that are appropriated by children in the course of their intellectual development (Vygotsky, 1978). It is important to stress that for Vygotsky, children's mastery of a counting system does merely enhance or amplify an already existing cognitive capability. He instead argued that children's ability to reason numerically is created as they appropriate the counting systems of their culture. This example illustrates Vygotsky's more general contention that children's minds are formed as they appropriate sign systems and other artifacts.

In the most well known series of investigations that he conducted, Vygotsky attempted to demonstrate the crucial role of face-to-face interactions in which an adult or more knowledgeable peer supports the child's use of an intellectual tool such as a counting system (Vygotsky, 1981). He concluded from these studies that the use of sign systems initially appears in children's cognitive development on what he termed the "intermental" plane of social interaction and that, over time, the child eventually becomes able to carry out what was previously a joint activity on his or her own. On this basis, he argued that the child's mind is created via a process of internalization from the intermental plane of social interaction to the "intramental" plane of individual thought.

The central role that Vygotsky attributed to social interactions with more knowledgeable others usually features prominently in accounts of his work. However, there is some indication that shortly before his premature death in 1934, he began to view the relation between social interaction and cognitive development as a special case of a more general relation between cultural practices and cognitive development (Davydov & Radzikhovskii, 1985; Minick, 1987). This aspect of sociocultural theory was developed more fully after Vygotsky's death by a group of Soviet psychologists, the most prominent of whom was Alexei Leont'ev.

Although Leont'ev (1978; 1981) acknowledged the importance of face-to-face interactions, he saw the encompassing cultural practices in which the child participates as constituting the broader context of his or her development. For example, Leont'ev might have viewed interactions in which a parent engages a child in activities that involve counting as an instance of the child's initial, supported participation in cultural practices that involve dealing with quantities. He argued that the child's progressive participation in specific cultural practices underlies the development of his or her thinking. Intellectual development was, for him, synonymous with the process by which the child becomes a full participant in particular cultural practices. Because he considered the cognitive capabilities that a child develops to be inseparable from the cultural practices that constitute the context of their development, he viewed those capabilities to be characteristics of the child-in-culture-practice rather than the child per se.

The second contribution that Leont'ev made to sociocultural theory concerns his analysis of material objects and events. Although Vygotsky brought sign systems and other cultural tools to the fore, he gave less attention to material reality. In building on Vygotsky's ideas, Leont'ev argued that material objects as they come to be experienced by the developing child are defined by the cultural practices in which he or she participates. For example, a pen becomes a writing instrument rather than a brute material object for the child as he or she participates in literacy practices. In Leont'ev's view, the child does not come into contact with material reality directly, but is instead oriented to this reality as he or she participates in cultural practices. He therefore concluded that the meanings that material objects come to have are a product of their inclusion in specific practices. This thesis serves to underscore his argument that the individual-in-cultural-practice constitutes the appropriate analytical unit.

To clarify the distinction between the individual as characterized by cognitive and by sociocultural theorists, it is useful to contrast the role attributed to tools and social interactions in the two perspectives. Sociocultural theorists have sometimes accused proponents of the cognitive tradition on which I have focused of adopting a so-called Robinson Crusoe perspective in which they study the reasoning of socially and culturally isolated people. It is therefore important to emphasize that adherents of the cognitive tradition readily acknowledge that students' reasoning is influenced by both the tools that they use to accomplish goals and by their ongoing social interactions with others. However, tools and others' actions are considered to be external to students' reasoning, and the focus on documenting how students' interpret them. Following Vygotsky, sociocultural theorists question the assumption that social process can be clearly partitioned off from cognitive processes and treated as external conditions for them. These theorists instead view cognition as extending out into the world and as being inherently social. They therefore attempt to break down a distinction that is basic to the cognitive perspective and indeed to experimental psychology, that between the reasoner and the world reasoned about. Furthermore, following Leont'ev, sociocultural theorists typically situate tool use and face-to-face interactions within encompassing cultural practices. From this perspective, students' actions are viewed as elements of a system of cultural practices and students are viewed as participating in cultural practices even when they are in physical isolation from others.

In considering the potential usefulness of sociocultural theory, I should acknowledge that the instructional designs that I and my colleagues have developed in recent years to support students' mathematical learning are broadly compatible with some of the basic tenets of this theoretical perspective (P. Cobb & McClain, 2002). Nonetheless, I contend that sociocultural theory is of limited utility when actually formulating designs at the classroom level. The contributions of Davydov (1988a, 1988b), notwithstanding, it is in fact difficult to identify instances of influential designs whose development has been primarily informed by sociocultural theory. This becomes understandable once we note that the notion of cultural practices employed by sociocultural theorists typically refers to ways of talking and reasoning that have emerged during extended periods of human history. This construct makes it possible to characterize mathematics as a complex human activity rather than as disembodied subject matter (van Oers, 1996). The task facing both the teacher and the instructional designer is therefore framed as that of supporting and organizing students' induction into practices that have emerged during the discipline's intellectual history. While the importance of the goals inherent in this framing is indisputable, they provide only the most global orientation for design. A key difficulty is that the disciplinary practices that are taken as the primary point of reference exist prior to and independently of the activities of teachers and their students. In contrast, the ways of reasoning and communicating that are actually established in the classroom do not exist independently of the teacher's and students' activity, but are instead constituted by them in the course of their ongoing interactions

(Bauersfeld, 1980; Beach, 1999; Boaler, 2000; P. Cobb, 2000). A central challenge of design is to develop, test, and refine conjectures about both the classroom processes in which students might participate and the nature of their mathematical learning as they do so. Sociocultural theory provides only limited guidance because the classroom processes on which design focuses are emergent phenomena rather than already-established practices into which students are inducted.

Extending our purview beyond the classroom, there are two areas of where, in my judgment, sociocultural theory can make significant contributions. I introduce each by first discussing recent bodies of scholarship developed within this tradition that are relevant to mathematics educators. The first body of scholarship is exemplified by investigations that have compared mathematical reasoning in school with that in various out-of-school settings such as grocery shopping (Lave, 1988), packing crates in a dairy (Scribner, 1984), selling candies on the street (Nunes, Schliemann, & Carraher, 1993; Saxe, 1991), playing dominoes and basketball (Nasir, 2002), laying carpet (Masingila, 1994), woodworking (Millroy, 1992), and sugar cane farming (de Abreu, 1995). These studies document that people develop significantly different forms of mathematical reasoning as they participate in different cultural practices that involve the use of different tools and sign systems, and that are organized by different overall motives (e.g., learning mathematics as an end in itself in school versus doing arithmetical calculations while selling candies on the street in order to survive economically). This approach of contrasting the forms of reasoning inherent in different cultural practices bears directly on issues of equity in students' access to significant mathematical ideas.

Although an adequate treatment of cultural diversity and equity is beyond the scope of this chapter, it is worth noting that a number of investigators have documented that the out-of-school practices in which students participate can involve differing norms of participation, language, and communication, some of which might be in conflict with those that the teacher seeks to establish in the mathematics classroom (Civil & Andrade, 2002; Ladson-Billings, 1998; Zevenbergen, 2000). An emerging line of research in mathematics education draws on sociocultural theory to document such conflicts and to understand the tensions that students experience (Boaler & Greeno, 2000; Gutiérrez, 2002; Martin, 2000; Moschkovich, 2002). The value of work of this type is that it enables us to view students' activity in the classroom as situated not merely with respect to the immediate learning environment, but with the respect to their history of participation in the practices of particular out-of-school groups and communities. It therefore has the potential to inform the development of designs in which the diversity in the out-of-school practices in which students participate is treated as an instructional resource rather than an obstacle to be overcome (Bouillion & Gomez, 2002; Civil, 2002; Gutstein, 2002; Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes, 2001).

The second relevant body of sociocultural research centers on the notion of a community of practice. In their overview of this line of research, Lave and Wenger (1991) clarify that they consider learning to be synonymous with the changes that occur in people's activity as they move from relatively peripheral participation to increasingly substantial participation in the practices of established communities. In doing so, they also argue that the tools used by community members carry a substantial portion of a community's intellectual heritage. Franke and Kazemi (2001) and Stein et al. (1998) have used Wenger's (1998) more recent formulation of these ideas to analyze teachers' learning as they are inducted into the practices of an established professional teaching community. However, mathematics education researchers are yet to exploit the full potential of the notion of community of practice in my view. This becomes apparent when we follow Wenger (1998) in noting that this notion brings together (a) theories of social structure that give primacy to institutions, norms and rules, and (b) theories of situated experience that give primacy to the dynamics of everyday existence and the local construction of interpersonal events. These two types of theories correspond to a dichotomy in the teacher education literature between analyses that focus on the structural or organizational features of schools and analyses that focus on the role of professional development in supporting teachers' reorganization of their instructional practices (Engestrom, 1998; Franke, Carpenter, Levi, & Fennema, 2001).

As Engestrom (1998) observes, the notion of community of practice has the potential to transcend this dichotomy in the literature by providing a unit of analysis that captures social structures that are within the scope of teachers' engagement as they develop and refine their instructional practices. Elsewhere, I and my colleagues have drawn heavily on this notion to develop an analytic approach for locating mathematics teachers' instructional practices within the institutional settings of the schools in which they work (P. Cobb, McClain, Lamberg, & Dean, 2003). The goal of research of this type is to gain insight into the processes by which teachers' instructional practices are partially constituted by the institutional setting of the schools in which they work. The potential contribution of such work is indicated by the substantiated finding that teachers' instructional practices are profoundly influenced by the institutional constraints that they attempt to satisfy, the formal and informal sources of assistance on which they draw, and the materials and resources that they use in their classroom practice (Ball & Cohen, 1996; C. A. Brown, Stein, & Forman, 1996; Nelson, 1999; Price & Ball, 1997; Senger, 1999; Stein & Brown, 1997). Analyses that document these affordances and constraints can inform the development of designs for supporting teachers' learning. In particular, they orient researchers and teacher educators to consider whether their collaborations with teachers should involve concerted attempts to bring about change in the institutional settings in which the teachers have developed and revised their instructional practices.

In summary, sociocultural theory characterizes the individual as a participant in established, historically evolving cultural practices. Thus, whereas the cognitive tradition that I discussed accounts for learning in terms of the epistemic individual's reorganization of its activity, sociocultural theory does so by documenting process by which people become increasingly substantial participants in various cultural practices. I have questioned the relevance of sociocultural theory to the development of designs at the classroom level but identified two other areas in which it can potentially make contributions. The first concerns equity in students' access to significant mathematical ideas and involves analyzing the out-of-school practices in which they participate. The second involves analyses of the institutional settings in which teachers develop and refine their instructional practices that can inform the development of designs for supporting their learning. In both these areas, the central issue is that of understanding how people deal with the tensions that they experience when different practices in which they participate are in conflict. In the first case, the conflicts are between the out-of-school practice in which students participate and those established in the mathematics classroom, whereas in the second the conflicts are between the practices of the schools in which teachers work and those established in a professional teaching community.

Distributed Cognition

As I noted, sociocultural theory has developed largely independently of western psychology and draws inspiration directly from the writings of Vygotsky and Leont'ev. Distributed cognition, in contrast, has developed in reaction to mainstream cognitive science and incorporates aspects of the Russian work. Mainstream cognitive science should be differentiated from the cognitive tradition on which I have focused in this chapter in that it involves the observer's rather than the actor's viewpoint and posits what Anderson (1983) terms cognitive behaviors that intervene between stimulus and observed response activity. Several of the most important contributors to distributed cognition such as John Seeley Brown (1989), Alan Collins (1992), and James Greeno (1997) in fact achieved initial prominence as mainstream cognitive scientists before substantially modifying their theoretical commitments. Whereas sociocultural theorists usually frame people's reasoning as acts of participation in relatively broad systems of cultural practices, distributed cognition theorists typically restrict their focus to the immediate physical, social, and symbolic environment. Empirical studies conducted within the distributed tradition therefore tend to involve detailed analysis of either specific people's or a small group's activity rather than analyses of people's participation in established cultural practices.

As I indicated when clarifying the claim that the notion of the individual is conceptually relative, the term distributed cognition is most closely associated with Roy Pea. Pea (1985, 1993) coined this term to emphasize that, in his view, cognition is distributed across minds, persons, and symbolic and physical environments. As he and other distributed cognition theorists make clear in their writings, this perspective directly challenges a foundational assumption of both mainstream cognitive science and of the cognitive tradition that I have discussed. This is the assumption that cognition is bounded by the skin and can be adequately accounted for solely in terms of internal processes. Distributed cognition theorists instead see cognition as extending out into the immediate environment such that the environment becomes a resource for reasoning. As a consequence, the individual is characterized in this tradition as an element of a reasoning system.

In developing to this position, distributed cognition theorists have been influenced by a number of studies conducted by sociocultural researchers, particularly those that compare people's reasoning in different settings. In one of the most frequently cited investigations, Scribner (1984) analyzed the reasoning of workers in a dairy as they filled orders by packing products into crates of different sizes. Her analysis revealed that the loaders did not perform purely mental calculations but instead used the structure of the crates as a resource in their reasoning. For example, if an order called for ten units of a particular product and six units were already in a crate that held twelve units, experienced loaders rarely subtracted six from ten to find how many additional units they needed. Instead, they might realize that an order of ten units would leave two slots in the crate empty and just know immediately from looking at the partially filled crate that four additional units are needed. As part of her analysis, Scribner demonstrated that the loaders developed strategies of this type as they went about their daily business of filling orders. For distributed cognition theorists, this indicates that the system that did the thinking was the loader in interaction with a crate. From this perspective, the loaders' reasoning is therefore treated as emergent relations between them and the immediate environment in which they worked.

Part of the reason that distributed cognition theorists attribute such significance to Scribner's study and to other investigations conducted by sociocultural researchers is that they capture what Hutchins (1995) refers to as cognition in the wild. This focus on people's reasoning as they engage in both everyday and workplace activities contrasts sharply with the traditional school-like tasks that are often used to investigate cognition. In addition to questioning whether people's reasoning on school-like tasks constitutes a viable set of cases from which to develop adequate accounts of cognition, several distributed cognition theorists have also critiqued current school instruction. In doing so, they have broaden their focus beyond mainstream cognitive science's traditional emphasis on the structure of particular tasks by drawing attention to the nature of the classroom activities within which the tasks take on meaning and significance for students.

J. S. Brown et al. (1989) developed one such critique by observing that school instruction typically aims to teach students abstract concepts and general skills on the assumption that students will be able to apply them directly in a wide range of settings. In challenging this assumption, they argue that the appropriate use of a concept or skill requires engagement in activities similar to those in which the concept or skill was developed and is actually used. In their view, the well-documented finding that most students do not develop widely applicable concepts and skills in school is attributable to the radical differences between classroom activities and those of both the disciplines and of everyday, out-of-school settings. They contend that successful students learn to meet the teacher's expectations by relying on specific features of classroom activities that are alien to activities in the other settings. In developing this explanation, Brown et al. treat the concepts and skills that students actually develop in school as relations between students and the material, social, and symbolic resources of the classroom environment.

It might be concluded from the two examples given thus far, those of the dairy workers and of students relying on what might be termed superficial cues in the classroom, that distributed cognition theorists do not address more sophisticated types of reasoning. Researchers working in this tradition have in fact analyzed a number of highly technical, work-related activities. The most noteworthy of these studies is, perhaps, Hutchins' (1995) analysis of the navigation team of a navel vessel as they brought their ship into harbor. In line with other investigations of this type, Hutchins argues that the entire navigation team and the artifacts it used constitutes the appropriate unit for a cognitive analysis. From the distributed perspective, it is this system of people and artifacts that did the navigating and over which cognition was distributed. In developing his analysis, Hutchins pays particular attention to the role of the artifacts as elements of this cognitive system. He argues, for example, that the cartographer has done much of the reasoning for the navigator who uses a map. This observation is characteristic of distributed analyses and implies that to understand a cognitive process, it is essential to understand how parts of that process have, in effect, been sedimented in tools and artifacts. Distributed cognition theorists therefore contend the environments of human thinking are thoroughly artificial. In their view, the cognitive resources that people exercise in particular environments are partially constituted by the cognitive resources with which they have populate those environments. As a consequence, the claim that artifacts do not merely serve to amplify cognitive process but instead reorganize them is a core tenet of the distributed cognition perspective (Dörfler, 1993; Pea, 1993).

In contrast to sociocultural theory, distributed cognition treats classroom processes as emergent phenomena rather than already-established practices into which students are inducted. For example, distributed theorists consider that aspects of the classroom learning environment such as classroom norms, discourse, and ways of using tools are constituted collectively by the teacher and students in the course of their ongoing interactions. In my judgment, the distributed perspective therefore has greater potential than sociocultural theory to inform the formulation of designs at the classroom level. A number of design research studies have in fact been conducted from this perspective (Bowers, Cobb, & McClain, 1999; Confrey & Smith, 1995; Fishman et al., 2004; Hershkowitz & Schwarz, 1999; Lehrer, Strom, & Confrey, 2002). In investigations of this type, researchers both develop designs to “engineer” novel forms of mathematical reasoning, and analyze the process of students’ learning in these designed learning environments together with the means by which that learning is supported (P. Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Collins, Joseph, & Bielaczyc, 2004; Confrey & Lachance, 2000; Design-Based Research Collaborative, 2003; Edelson, 2002; Gravemeijer, 1994b). I noted that designs developed from the cognitive perspective on which I have focused typically emphasize the development of instructional activities. In contrast, distributed cognition theorists’ characterization of students as elements of reasoning systems orients them to construe the means of supporting students’ mathematical learning more broadly. The means of support that are incorporated into designs are usually not limited to instructional activities but also encompass classroom norms, the nature of discourse, and the ways in which notations and other types of tools are used. Thus, whereas design from the cognitive perspective involves developing instructional activities as informed by analyses of specific students’ reasoning, design from the distributed perspective focuses on the physical, social, and symbolic classroom environment that constitutes the immediate situation of the students’ mathematical learning.

Given my generally positive assessment of the usefulness of the distributed perspective in informing the development, testing, and revision of designs, it is also important to note two potential limitations. The first limitation concerns the scant attention typically given to issues of equity. For example, the focus of researchers who develop and refine designs and the classroom level usually centers on students’ individual and collective development of particular forms of mathematical reasoning. Pragmatically, it is essential that students come to see classroom activities as worthy of their engagement if the designs are to be effective. However, the process of supporting students’ engagement by cultivating their mathematical interests is rarely an explicit focus of inquiry (for exceptions, see diSessa, 2001; Eisenhart & Edwards, 2001, April). As a consequence, differences in students’ engagement that might reflect differential access to the instructional activities used and to the types of discourse established in the classroom can easily escape notice. In my view, this limitation stems from an almost exclusive focus on the classroom as the immediate context of students’ learning. This focus precludes a consideration of tensions that some students might experience between aspects of this social context and the out-of-school practices in which they participate. Adherents to this perspective can address this limitation by coordinating their viewpoint with a sociocultural perspective that enables them to see students’ classroom activity as situated not merely with respect to the immediate learning environment, but also with respect to their history of participation in the practices of out-of-school groups and communities.

The second limitation concerns the manner in which the characterization of the individual as an element of a reasoning system is taken as delegitimizing cognitive analysis of specific students’ reasoning. As I indicated earlier in this chapter, this view of the individual reflects the evolution of the distributed perspective from mainstream cognitive science in that it involves equipping the individual as portrayed by this latter perspective with cultural tools and placing it in social context. I consider this restriction to be a limitation of the distributed perspective because it fails to acknowledge the contributions that cognitive analyses of specific students’ reasoning can make to the process of adjusting and modifying an instructional design (see P. Cobb, 1998, for a detailed discussion of this point). Furthermore, in the hands of a skillful teacher, the diversity in students’ reasoning is a primary resource on which he or she can draw to support sustained classroom discussions that focus on substantive mathematical issues. Earlier in this chapter, I followed Putnam (1987) in endorsing Quine’s (1953) argument that we should accept the existence of abstract mathematical entities on the grounds that this ontology is indispensable to the mathematics viewed as a practice. Similarly, I contend that we should accept the existence of specific students’ mathematical reasoning because this ontology is indispensable to design and teaching in mathematics education. It is therefore necessary, in my view, to resist theoretical arguments that delegitimize this ontology. In the next section of this chapter, I indicate how it might be possible to capitalize on the potential contributions of the distributed perspective while circumventing this limitation.

In summary, the distributed perspective emerged in response to the limited attention given to context, culture, and affect by mainstream cognitive science (De Corte et al., 1996). The individual is characterized as an element of a reasoning system that also includes aspects of the immediate physical, social, and symbolic environment. In contrast to sociocultural theorists’ focus on people’s participation in established cultural practices, distributed theorists usually conduct detailed analyses of reasoning processes that are stretched over people and the aspect of their immediate environment that they use as cognitive resources. In my view, the distributed perspective has greater potential than sociocultural theory to contribute to the formulation of designs at the classroom level because it treats classroom processes as emergent phenomena. However, I tempered this positive appraisal by noting two limitations. These concern the limited attention given to issues of equity and the disavowal of cognitive analyses of specific students' mathematical reasoning. Taken together, these two limitations indicate the value of attempting to capitalize on the ways that multiple perspectives can contribution to the enterprise of developing, testing, and revising designs for supporting learning. I address this issue shortly.

REFLECTION

The contrasts I have drawn between the four theoretical perspectives are summarized in Table 1. The development of these contrasts involved the use of two criteria of comparison. The first criterion concerns how each perspective characterizes the individual whereas the second focused on the potential of each to contribute to our understanding of learning processes and the means of supporting their realization. Following Kuhn (1962; 1970), it is important to acknowledge that these criteria are not neutral standards but are instead values that I propose should be considered when coming to terms with the multiplicity of theoretical perspectives that characterize mathematics education research. As I indicated, these criteria reflect commitments and interests inherent in the view that mathematics education is a design science and, for this reason, are eminently debatable and are open to critique and revision. Furthermore, the process of using them to compare theoretical perspectives necessarily involves interpretation and judgment. Consequently, mathematics educators who use these same criteria "may nevertheless make different choices in the same concrete situation" (Kuhn, 1970, p. 262). My intent in proposing criteria and illustrating how they might be used has therefore not been to shut down debate in the face of competing theoretical perspectives, but to move the debate to a meta-level at which we are obliged to give good reasons for our theoretical choices. At this meta-level, the ideological fervor with which particular perspectives are sometimes promoted is no substitute for justifications that articulate the choice criteria or values together with the interests and concerns that they reflect. The pragmatic realist stance that I have taken to the issue of multiple theoretical perspectives will have achieved what Rorty (1979) refers to as its therapeutic purpose to the extent that justifications of this sort become commonplace.

THEORIZING AS BRICOLAGE

As part of the process of discussing the four theoretical perspectives, I briefly outlined the historical origins of each. I noted, for example, that experimental psychology has been profoundly shaped by its alliance with educational administrators. In contrast, the origins of the cognitive tradition on which I focused can be traced to Piaget's interest in problems of genetic epistemology. Sociocultural theory for its part reflects Vygotsky's and Leont'ev's commitment to the notion of the "the new socialist man" and to the view of education as a primary means of bringing about this change in Soviet society (van der Veer & Valsiner, 1991).

Table 1.1 Contrasts Between Four Theoretical Perspectives

Theoretical perspective	Characterization of the individual	Usefulness	Limitations
Experimental Psychology	Statistically constructed collective individual	Administration of educational systems	Limited relevance to design at classroom level
Cognitive Psychology	Epistemic individual as reorganizer of activity	Specification of "big ideas" Design of instructional activities	Means of supporting learning limited to instructional tasks
Sociocultural Theory	Individual as participant in cultural practices	Designs that take account of students' out-of-school practices Designs that take account of institutional setting of teaching and learning	Limited relevance to design at classroom level
Distributed Cognition	Individual element of a reasoning system	Design of classroom learning environments including norms, discourse, and tools	Delegitimizes cognitive analyses of specific students' reasoning

Finally, distributed cognition emerged in reaction to perceived shortcomings of mainstream cognitive science. The concerns and interests that motivated the development of each of these perspectives differ significantly from those of mathematics educators. In terms of the metaphor of theoretical perspectives as conceptual tools that I introduced earlier in this chapter, each of these perspectives is a tool that has been fashioned while addressing problems that are not of immediate concern to most mathematics educators. It is therefore unreasonable to expect that any one of these perspectives is ready-made for the collective enterprise of developing, testing, and revising designs. Given the limitations that I have discussed of each perspective with respect to this enterprise, the question that arises is not that of how to choose between the various perspectives. Instead, it is how they can be adapted to the concerns and interests of mathematics educators.

In addressing this question, I propose to view the four perspectives as sources of ideas that we can appropriate and modify for our purposes as mathematics educators. This process of developing conceptual tools for mathematics education research parallels that of instructional design as described by Gravemeijer (1994b).

> [Design] resembles the thinking process that Lawler (1985) characterizes by the French word *bricolage,* a metaphor taken from Claude Levi–Strauss. A *bricoleur* is a handy man who invents pragmatic solutions in practical situations. . . . [T]he bricoleur has become adept at using whatever is available. The bricoleur's tools and materials are very heterogeneous: Some remain from earlier jobs, others have been collected with a certain project in mind. (p. 447)

Similarly, I suggest that rather than adhering to one particular theoretical perspective, we act as bricoleurs by adapting ideas from a range of theoretical sources.

To illustrate this approach, I take as a case an interpretive framework that several colleagues and I developed over a number of years while addressing concrete problems and issues that arose while working in classrooms (P. Cobb, Stephan, McClain, & Gravemeijer, 2001; P. Cobb & Yackel, 1996). The intent of the framework is to locate students' mathematical reasoning in the social context of the classroom in a manner that can feed back to inform instructional design and teaching. For my current purposes, it suffices to note that the framework involves the coordination of two distinct perspectives on classroom activity. One is a social perspective that is concerned with ways of acting, reasoning, and arguing that have been established as normative in a classroom community. From this perspective, an individual student's reasoning is framed as an act of participation in these normative activities. The other is a cognitive perspective that focuses squarely on the nature of individual students' reasoning or, in other words, on their specific ways of participating in communal classroom activities. Analyses developed by using the framework bring the diversity in students' mathematical reasoning to the fore while situating that diversity in the social context of their participation in communal activities.

We take the relation between the social and cognitive perspectives to be one of reflexivity. This is an extremely strong relationship that does not merely mean that the two perspectives are interdependent. Instead, it implies that neither exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective (Mehan & Wood, 1975). For example, the collective activities of the classroom community (social perspective) emerge and are continually regenerated by the teacher and students as they interpret and respond to each other's actions (cognitive perspective). Conversely, the teacher's and students' interpretations and actions in the classroom (cognitive perspective) are not seen to exist apart from their participation in communal classroom practices (social perspective). The coordination is therefore not between individual students and the classroom community viewed as separate entities but between two alternative ways of looking at and making sense of what is going on in classrooms.

This interpretive framework is a bricolage in that the social perspective draws on sociocultural theory (Cole, 1996; Lave, 1991; Rogoff, 1990) and the cognitive perspective draws on both cognitive psychology (Piaget, 1970; Steffe & Kieren, 1994; Thompson, 1991), and distributed accounts of cognition (e.g., Hutchins, 1995; Pea, 1993; Wertsch, 1998, 2002). One of the key theoretical constructs that we use when we take a social perspective, that of a classroom mathematical practice, serves to illustrate how we have appropriated ideas to our agenda as mathematics educators. We developed this construct by adapting sociocultural theorists' notion of a cultural practice. As I indicated when discussing sociocultural theory, this idea is attractive because it makes it possible to characterize mathematics as a complex human activity rather than as disembodied subject matter. However, I also noted that the notion of a practice as framed in sociocultural theory is problematic from the point of view of design because practices are typically characterized as existing prior to and independently of the teacher's and students' activity. We therefore modified this notion by explicitly defining a classroom mathematical practice as an emergent phenomenon that is established jointly by the teacher and students in the course of their ongoing interactions. This is a non-trivial adaptation in that students are then seen to contribute to the development of the classroom norms and practices that constitute the social situation of their mathematical learning.

We also made modifications when fashioning a cognitive perspective appropriate for our purposes by adapting ideas from the cognitive perspective I have discussed and from distributed cognition. For example, we took from the cognitive perspective the notion of learning as a process of reorganizing activity. However, influenced by distributed theories of intelligence, we found it important to broaden our view of activity so that it is not restricted solely to solo sensory-motor and conceptual activity but instead reaches out into the world and includes the use of tools and symbols. The rationale for this modification is at least in part pragmatic in that our work as instructional designers involves developing notation systems, physical tools, and computer-based tools for students to use. We therefore needed an analytic approach that can take account both of the diverse ways in which students reason with tools and symbols, and of how those ways of reasoning evolve over time.

Although we saw value in distributed accounts of intelligence, we could not accept this theoretical orientation ready-made given its rejection of analytical approaches that focus explicitly on the nature of specific students' reasoning. The adaptation we made was to modify how the individual is characterized. As I have indicated, distributed theorists appear to have accepted the portrayal of the individual offered by mainstream cognitive science and propose equipping this character with cultural tools and locating it in social context. The primary adaptation we made was to characterize the individual not as needing to be placed in its immediate physical, social, and symbolic environment, but as already acting in that environment. The tools and symbols that students use are then viewed as constituent parts of their activity rather than as standing apart from or outside their activity.

Once this modification is made, what is viewed as a student-tool system from the perspective of distributed cognition becomes, in the psychological perspective we take, an individual student engaging in mathematical activity that involves *reasoning with* tools and symbols. Thus, although the focus of this psychological perspective is explicitly on the quality of individual students' reasoning, its emphasis on tools is generally consistent with the notion of mediated action as discussed by sociocultural theorists such as Wertsch (2002). Further, as I have indicated, the remaining component of the functional system posited by distributed theorists, the classroom social context, becomes an explicit focus of attention when this psychological perspective is coordinated with the social perspective. The interpretive framework therefore characterizes students as reasoning with tools while participating in and contributing to the development of communal practices. With regard to usefulness, the framework yields analyses of students' mathematical learning that are tied to the classroom social setting in which that learning actually occurs. As a consequence, these analyses enable us to tease out aspects of this setting that served to support the development of students' reasoning. This, in turn, makes it possible to develop testable conjectures about ways in which those means of support and thus the instructional design can be improved.

In keeping with the critical stance I took to the four theoretical perspectives, I should acknowledge that the interpretive framework has at least two major limitations. These concern the isolation of classroom learning environments from the institutional settings of the schools in which they are located, and failure to address issues of cultural diversity and equity in students' access to significant mathematical ideas. As a consequence, these two areas have a become a focus of our research in recent years (P. Cobb & Hodge, 2002; P. Cobb, McClain et al., 2003). It is also worth noting that the framework does not draw on the theoretical perspective of experimental psychology. This is primarily because experimental psychology views the classroom learning environment from an administrative viewpoint and characterizes it as composed of independent variables that can be manipulated from the outside. In contrast, the framework I have outlined characterizes the classroom learning environment from the inside as jointly constituted by the teacher and students. Analyses developed when using this framework will therefore be of little value to most administrators. The experimental psychology perspective might be useful when the goal is to contribute to public policy discourse about mathematics teaching and learning.

My purpose in discussing the interpretive framework has been to illustrate the process of adapting and modifying ideas appropriated from a range of theoretical sources. The pragmatic spirit of the bricolage metaphor indicates that the goal in doing so is to fashion conceptual tools that are useful for our purposes as mathematics educators. The metaphor therefore serves to differentiate relatively modest efforts of the type that I have illustrated from more ambitious projects that aim to develop theoretical cosmologies (Shotter, 1995). For example, our goal in developing the framework was to craft a tool that would enable us to make sense of what is happening in mathematics classrooms rather than to produce a grand synthesis of cognitive psychology, sociocultural theory, and distributed cognition. In my view, theorizing as a modest process of bricolage offers a better prospect of mathematics education research developing an intellectual identity distinct from the various perspectives on which it draws than does the attempt to formulate all-encompassing theoretical schemes.

INCOMMENSURABILITY

The approach I have illustrated for comparing theoretical perspectives is best viewed as a proposal about how we might cope with competing theoretical perspectives in mathematics education research. The account that I have given of the relations between different perspectives can be contrasted with analyses that portray a historical sequence of theoretical perspectives, each of which overcomes the limitations of its predecessors. Accounts of this latter type depict the history of a field as an ordered progression that typically culminates with the perspective to which the writer subscribes. As Guerra (1998) notes, these narratives are based on the implicit metaphor of theoretical developments as a *relentless march of progress.* For example, adherents of distributed cognition frequently portray their perspective as overcoming the limitations of mainstream cognitive science. Proponents of perspectives that have supposedly been superseded typically employ a different type of narrative that is based on the metaphor of *potential redemption.* For example, experimental psychologists frequently characterize the increasing prominence of competing perspectives as a period in which the issue of what counts as credible evidence has been largely ignored, and portray their perspective as offering redemption by making educational research a prestigious scientific enterprise.

The account I have given reflects an alternative metaphor, that of *co-existence and conflict.* The tension between the march of progress and potential redemption narratives indicates the relevance of this metaphor. I fleshed out this metaphor by proposing two criteria for comparing and contrasting competing perspectives. The first focused on how each perspective characterized the individual, thereby delineating the types of phenomena that proponents of the different perspectives are investigating. The second focused on the potential contributions of each perspective to the collective enterprise of formulating, testing, and revising designs for supporting learning. In developing this second criterion, I followed Dewey (1890/1969), Pepper (1942), and Prawat (1995) in arguing that a theoretical perspective is a useful conceptual tool for mathematics educators to the extent that it gives insight and understanding into learning processes and the specific means of supporting their realization. In using these two criteria, I noted that the goals for which each of the four perspectives was originally developed differ from those of mathematics educators. On this basis, I argued that we should view the various co-existing perspectives as sources of ideas to be adapted to our purposes.

The contrasting ways in which the different perspectives characterize the individual indicate that they are incommensurable. In terms of Putnam's (1987) pragmatic realism, adherents to the differing perspectives ask different types of questions and produce different forms of knowledge as they attempt to develop insights into different realities. I followed Putnam (1987) and Kuhn (1962, 1977) in arguing the realities that researchers investigate are conceptually relative to their particular theoretical perspectives, but rejected the claim that any of these realities is as good as any other. The approach I took is consistent with Feyerabend's (1975) claim that we cope with immensurability both in research and in other areas of life by drawing comparisons and contrasts in the course of which we delineate similarities and differences. Feyerabend also argued that there is no single ultimate grid for comparing theoretical perspectives, and demonstrated that they can be compared in multiple ways. The primary challenge posed by incommensurability is to develop a way of comparing and understanding different perspectives. It was for this reason that I discussed the two criteria I used in some detail.

As Bernstein (1983) observed, the process of comparing immensurable perspectives has parallels with anthropology in that the goal is to figure out what the "natives" think they are doing (Geertz, 1973). Kuhn (1977) described how he attempts to avoid merely imposing his viewpoint when he discussed his work as a philosopher and historian of science.

> [The] plasticity of texts does not place all ways of reading on a par, for some of them (ultimately, one hopes, only one) possess a plausibility and coherence absent from others. Trying to transmit such lessons to students, I offer them a maxim: When reading the works of an [historically] important thinker, look first for the apparent absurdities in the text and ask yourself how a sensible person could have written them. When

> you find an answer, I continue, when those passages make sense, then you may find that more central passages, ones you previously thought you understood, have changed their meaning. (p. xii)

The openness inherent in this stance to incommensurability has the benefit that in coming to understand what adherents to an alternative perspective think they are doing, we develop a more sensitive and critical understanding of some of the taken-for-granted aspects of our own perspective. This understanding is critical to the justification of theoretical choices in that these justifications should, in my view, involve the specification of both the choice criteria used and the interests and concerns that they reflect.

As Geertz (1973) emphasizes, the absence of abstract, cross-cultural universals does not condemn anthropology to absolute relativism.

> If we want to discover what man amounts to, we can only find it in what men are, and what men are, above all other things, is various. It is in understanding that variousness—its range, its nature, its basis, and its implications—that we shall come to construct a concept of human nature that more than a statistical shadow and less than a primitive dream has both substance and truth. (p. 52)

Similarly, the absence of a neutral framework for comparing incommensurable theoretical perspectives does not condemn us to absolute relativism in which theoretical decisions amount to nothing more than personal whim or taste. A primary purpose of philosophy for Dewey, Gadamer, Kuhn, Putnam, and Rorty is to transcend the dichotomy between an unobtainable neutral framework on the one hand and an absolute, "anything goes" brand of relativism on the other. Philosophy as they conceive it is a discourse about incommensurable perspectives and discourses. Its intent is both edifying and therapeutic in that it aims to support conversation both about and between various perspectives. My purpose in this chapter has been to advance a conversation of this type in the mathematics education research community.

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